سوال ۱:

$$\log_{nm} m^2 n = \frac{\log_n m^2 n}{\log_n nm} = \frac{\log_n m^2 + \log_n n}{\log_n m + \log_n n} = \frac{2\log_n m + 1}{\log_n m + 1} = \frac{2a+1}{a+1} = b$$

$$[b] = \left[\frac{a+a+1}{a+1}\right] = \left[\frac{a}{a+1}+1\right] = \left[\frac{a}{a+1}\right]+1 \xrightarrow{a>0} [b] = 0+1 = 1$$

سوال ۲:

الف) $y = \sqrt{\dfrac{x}{\log_{\frac{1}{2}} x}}$ ‏ $x>0$

$$\frac{x}{\log_{\frac{1}{2}} x} \geq 0 \xrightarrow{\text{با هم علامت باشند}} \log_{\frac{1}{2}} x ,\ x \Rightarrow \log_{\frac{1}{2}} x > 0 \Rightarrow x<1 \Rightarrow D_f = (0,1)$$

ب) $y = \dfrac{\log_4 (x^2-x-2)}{\sqrt{x^2-1}+1}$

$$x^2-1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow x \geq 1 \quad x \leq -1 \quad (1)$$

$$x^2-x-2>0 \Rightarrow (x-2)(x+1)>0 \Rightarrow \quad (1),(2) \longrightarrow D_f = (-\infty,-1)\cup(2,+\infty)$$

	-1		2	
$+$		$-$		$+$

$x<-1$, $x>2$ (2)

سوال ۳:

$$2\log_x a + \log_a \sqrt{x} = 2 \xrightarrow{x=9} 2\log_9 a \to 3^2 = \frac{1}{2}\log_3 a = \frac{1}{2\log_a 3} \Rightarrow \cancel{2}\times\frac{1}{\cancel{2}\log_a 3} + \log_a 3 = 2 \xrightarrow{\log_a 3 = t}$$

$$t + \frac{1}{t} = 2 \Rightarrow \frac{t^2+1}{t} = 2 \Rightarrow t^2-2t+1=0$$

$$(t-1)^2 = 0 \Rightarrow t=1 \Rightarrow \log_a 3 = 1 \Rightarrow \boxed{a=3}$$

سوال ۴:

$$\log\frac{5}{3} = \log 5 - \log 3 = 0{,}7 - 0{,}4 = 0{,}3$$

$$\log\frac{15}{2} = \log 10 - \log 2 = 1 - 0{,}3 = 0{,}7$$

$$\log 9 \to 3^2 = 2\log 3 = 2\times 0{,}4 = 0{,}8$$

$$\log 15 \to 5\times 3 = \log 5 + \log 3 = 0{,}7 + 0{,}4 = 1{,}1$$

$$\Rightarrow 0{,}3x^2 + 0{,}8x - 1{,}1 = 0 \xrightarrow{\times 10} 3x^2 + 8x - 11 = 0 \xrightarrow{\text{ضرب}} x^2 + 8x - 33 = 0$$

$$(x+11)(x-3) = 0 \Rightarrow x = \frac{3}{3} = 1, \quad x = -\frac{11}{3}$$

$$\text{اختلاف} = 1 - \left(-\frac{11}{3}\right) = 1 + \frac{11}{3} = \boxed{\frac{14}{3}}$$

سوال ۵:

$$\log_5 2 = \frac{\log_2 2}{\log_2 5} = \frac{1}{\log_2 5} = \frac{1}{2} \Rightarrow \log_2 5 = 2$$

$$\log_{14} 10 = \frac{\log_2 10}{\log_2 14} = \frac{\log_2 5 + \log_2 2}{\log_2 7 + \log_2 2} = \frac{2+1}{2{,}8+1} = \frac{3}{3{,}8} = \boxed{\frac{15}{19}}$$

$\log_2 3 = \frac{1}{\log_3 2} = 1.6 \Rightarrow \log_3 2 = \frac{5}{8}$ $\quad \log_{10} 6 = \frac{\log_3 6}{\log_3 10} = \frac{\log_3 2 + \log_3 3}{\log_3 5 + \log_3 2} = \frac{\frac{5}{8}+1}{1.5+1} = \frac{\frac{13}{8}}{\frac{5}{2}} = \frac{13}{20} = \boxed{0.65}$:س۶

$\log_{\sqrt[3]{4} \to 4^{\frac{2}{3}}} 18 = m \Rightarrow \frac{3}{2}\log_4 18 = m \Rightarrow \log_4 \overset{2\times3\times3 \leftarrow}{18} = \frac{2}{3} m$ $\quad \cancel{\log_4 2}\ \frac{1}{2} + 2\log_4 3 = \frac{2}{3} m \Rightarrow 2\log_4 3 = \frac{2m-1}{2} \Rightarrow \log_4 3 = \frac{2m-1}{4}$:س۷

$\log_4 (12 \to 4\times3) = \log_4 4 + \log_4 3 = 1 + \frac{2m-1}{4} = \boxed{\frac{2m+3}{4}}$

$(0.4)^{2x-1} = (2\times5)^{2x-1} = 2^{2x-1} \times 5^{1-2x}$ $\quad \left(\frac{125}{8}\right)^{x^2} = (5^3 \times 2^{-3})^{x^2} = 5^{3x^2} \times 2^{-3x^2} \Rightarrow 2x-1 = -3x^2 \Rightarrow 3x^2+2x-1=0$:س۸

$\Rightarrow \underset{\text{غ ق ق}}{x=-1},\ x = \frac{1}{3}$ $\quad 9x+1>0 \Rightarrow 9x>-1 \Rightarrow x > \frac{-1}{9}$ $\quad \log_8 (9x+1) = \log_{8\to2^3} (4\to2^2) = \boxed{\frac{2}{3}}$

$\log_8 b = \frac{1}{3}\log_2 b = \frac{2}{3}(1+a) \Rightarrow \log_2 b = \underset{\log_2 4}{2} + \underset{\to 2\log_2 3}{2a} = \log_2 (4\times3\times3) = \log_2 36$:س۹

$\Rightarrow b = 36 \Rightarrow \sqrt{b} - 8 = 1 - \log 10 = \boxed{2}$

$-4ax^2 + bx + \frac{1}{2}c = 0 \quad S = \frac{-b}{a} = \frac{-b}{-4a} = \frac{b}{4a}$ $\quad \log_{10} 4 = \frac{4a}{b} \Rightarrow \log_{10} 2 = \frac{2a}{b}$:س۱۰

$a = \frac{b+c}{2} \Rightarrow 2a = b+c \Rightarrow c = 2a-b$

$\frac{c}{a} = \frac{2a-b}{a} = 2 - \frac{b}{a} = 2 - 2\log_2 10 = \left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}} = \left(\frac{1}{\sqrt[8]{2}}\right)^{2-2\log_2 10} = \left(2^{-\frac{1}{8}}\right)^{2-\log_2 10} = \frac{\left(2^{-\frac{1}{8}}\right)^2}{\left(2^{-\frac{1}{8}}\right)^{\log_2 10}} =$

$\frac{2^{-\frac{1}{4}}}{10^{-\frac{1}{8}}} = (0.2)^{-\frac{1}{4}} = 5^{\frac{1}{4}}$

$= \sqrt[4]{5}$