Subject .... Date ....

ستاره شش حسابان یازدهم دفترچه

① $\log_n m = a \qquad \log_{mn} m^2 n = b \qquad a>0 \qquad [b]=?$

$n^a = m$

$b = \log_{n^a \times n} n^{2a}\times n = \log_{n^{a+1}} n^{2a+1} \to \frac{2a+1}{a+1}\log_n n \to b = \left[\frac{a+a+1}{a+1}\right]$

$\Rightarrow b = 1 + \left[\frac{a}{a+1}\right] = 1$

---

② الف) $y = \sqrt{\frac{x}{\log_{\frac{1}{2}} x}} \to x>0$

$\log_{\frac12} x \neq 0 \to x \neq 1 \qquad D_f = (0,1)$

	0		1	
		+		−

ب) $\frac{\log_4 (x^2-x-2)}{\sqrt{x^2-1}+1}$

$x^2 - x - 2 > 0 \qquad x=-1,\ x=2$

$x^2-1 \geq 0 \to x^2 \geq 1 \to x \geq 1,\ x \leq -1$

$\sqrt{x^2-1} \neq -1$ ✓

$\Rightarrow D_f = (-\infty,-1)\cup(2,+\infty)$

---

③ $2\log_x a + \log_a \sqrt{x} = 2 \qquad x = 9 \qquad a = ?$

$2\log_x a + \frac{1}{2}\log_a x = 2 \longrightarrow 2\log_x a + 2\log_x a = 2 \to \log_x a = \frac{1}{2}$

$a = \sqrt{9} = 3 \longleftarrow a = x^{\frac12} = \sqrt{x}$

---

④ $\log 2 = 0.3 \qquad \log 3 \simeq 0.4 \qquad (\log\frac{5}{3})x^2 + (\log 9)x - \log 15 = 0$

$(\log 5 - \log 3)x^2 + (2\log 3)x - (\log 3 + \log 5) = 0 \longrightarrow$ جمع ضرایب = صفر

$x = 1$

$x = \frac{c}{a}$

$\frac{c}{a} = \frac{-\log 3 - \log 5}{\log 5 - \log 3} = \frac{-\log 3 - \log 10 + \log 2}{\log 10 - \log 2 - \log 3} = \frac{-1.1}{0.3} = \frac{-11}{3}$

$x_2 - x_1 = 1 - (-\frac{11}{3}) = \frac{14}{3}$

---

⑤ $\log_2 7 = 2.8 \qquad \log_5 2 = 0.5 \qquad \log_{14} 10 = ?$

$\log_5 2 \times \log_2 7 = 0.5 \times 2.8 \Rightarrow \log_5 7 = 1.4 \qquad \log_{14} 10 = \frac{\log_5 10}{\log_5 14}$

$\log_{14} 10 = \frac{\log_5 2 + \log_5 5}{\log_5 2 + \log_5 7} = \frac{\overset{1.5}{0.5+1}}{\underset{1.9}{0.5+1.4}} = \frac{15}{19}$

---

⑥ $\log_3 5 = 1.5 \qquad \log_2 3 = 1.6 \qquad \log_{15} 6 = ?$

$\log_3 5 \times \log_2 3 = \log_2 5 = 2.4$

$\log_{15} 6 = \frac{\log_2 6}{\log_2 15} = \frac{\overset{1}{\cancel{\log_2 2}} + \overset{1.6}{\log_2 3}}{\underset{1.6}{\cancel{\log_2 3}} + \underset{2.4}{\log_2 5}} = \frac{2.6}{4} = \boxed{0.65}$

$\log_8 18 = m \qquad \log_4 12 = ?$

(7)

$\log_4 12 = \dfrac{\log_8 12}{\log_8 4} \longrightarrow \log_8 12 = \log_8 4 + \log_8 3 = \frac{2}{3} + \frac{1}{3}\log_2 3$

$\log_8 4 = \frac{2}{3}\log_2 2 = \frac{2}{3}$

$\log_8 18 = m \longrightarrow \log_8 9 + \log_8 2 = m \longrightarrow \frac{2}{3}\log_2 3 + \frac{1}{3}\log_2 2 = m \to \frac{2}{3}\log_2 3 + \frac{1}{3} = m$

$\log_2 3 = \dfrac{3m-1}{2} \qquad \log_4 12 = \dfrac{\frac{2}{3} + \frac{1}{3}\log_2 3}{\frac{2}{3}} = \dfrac{\frac{2}{3} + \frac{1}{3}\left(\frac{3m-1}{2}\right)}{\frac{2}{3}} = \dfrac{4+3m-1}{4}$

$= \frac{3}{4}(m+1)$

---

(8)

$\log_8 4a+1 \qquad (0.4)^{2a-1} = \left(\frac{125}{8}\right)^{a^2}$

$\left(\frac{4}{10}\right)^{2a-1} = \left(\frac{5^3}{2^3}\right)^{a^2} \Rightarrow \left(\frac{2}{5}\right)^{2a-1} = \left(\frac{5}{2}\right)^{3a^2} \longrightarrow 2a-1 = -3a^2$

$3a^2 + 2a - 1 = 0 \longrightarrow a^2 + 2a - 3 = 0 \to a = -1$ غ ق ق

$\log_{2^3}(3+1) = \log_{2^3} 2^2 = \frac{2}{3}\log_2 2 = \frac{2}{3} \qquad a = \frac{1}{3}$ ق ق

---

(9)

$\log_2 3 = a \qquad \log_8 b = \frac{2}{3}(1+a) \qquad \log(3b-8) = ?$

$\log_{2^3} b = \frac{1}{3}\log_2 b = \frac{2}{3}(1+a) \longrightarrow \log_2 b = 2 + 2a$

$2^2 \times 2^{2a} = b \longrightarrow 4 \times 9^{\log_2 3}... = 36 = b \qquad \log(3 \times 36 - 8) = \log 100 = 2$

---

(10)

$-\frac{1}{2}ax^2 + bx + \frac{1}{2}c = 0 \qquad S = \dfrac{-b}{-4a} = \dfrac{b}{4a} = \log 4$

$\dfrac{a}{b} = \dfrac{\log 4}{4} = \dfrac{2\log 2}{2}$

$2a = b + c \longrightarrow \dfrac{2a}{b} = 1 + \dfrac{c}{b} \longrightarrow 2\left(\dfrac{\log 2}{2}\right) = 1 + \dfrac{c}{b}$

$\dfrac{c}{b} = -1 + \log 2 \longrightarrow \dfrac{c}{a} = \dfrac{-1 + \log 2}{\frac{\log 2}{2}} \Rightarrow \dfrac{c}{a} = \dfrac{\log 2 - \log 10}{\frac{1}{2}\log 2}$

$= \dfrac{\log \frac{1}{5}}{\frac{1}{2}\log 2} = \log_{\sqrt{2}} \frac{1}{5} \longrightarrow \left(\frac{1}{5}\right)^{-\frac{1}{2}} = \sqrt{5}$

$\log \sqrt{2}$