

$$\log_m n = a \rightarrow m = n^a$$

$$\log_{mn} m^n = \log_{mn} m^{n \times m} = \log_{mn} m^n + \log_{mn} m = b \rightarrow 1 + \log_{mn} m^n = b$$

$$\rightarrow b = \frac{a}{a+1} + 1 = \frac{a+1}{a+1} \quad a > 0 \quad 1 < b < 2$$

$$\rightarrow \lfloor b \rfloor = 1$$

الف) $y = \sqrt{\frac{n}{\log \frac{n}{r}}}$ $n > 0$ $\frac{n}{\log \frac{n}{r}} > 0$ $\frac{0}{-0+0-}$ $\rightarrow D = (0, 1)$

ب) $y = \frac{\log_r (n^2 - n - 2)}{\sqrt{n^2 - 1} + 1}$ $n^2 - n - 2 > 0$ $\frac{-1}{+4-0+}$ $\frac{2}{-}$ $\rightarrow D = (-\infty, -1) \cup (2, +\infty)$

$n^2 - 1 > 0 \rightarrow n^2 > 1 \rightarrow n > 1 \vee n < -1$
 $\sqrt{n^2 - 1} + 1 \neq 0 \quad \checkmark$

$$r \log_m a + \log_a \sqrt{m} = 2 \rightarrow \frac{r}{\log_m a} + \log_a \sqrt{m} = 2 \rightarrow \frac{r}{\log_m a} + \log_a \sqrt{m} = 2$$

$$\log_a \sqrt{m} = t \rightarrow \frac{1}{t} + t = 2 \rightarrow t^2 - 2t + 1 = 0 \rightarrow (t-1)^2 = 0 \rightarrow t = 1$$

$$\rightarrow \log_a \sqrt{m} = 1 \xrightarrow{m=9} \log_a 3 = 1 \rightarrow a' = 3 \rightarrow \boxed{a=3}$$

$$\log_r 3 = 0.5 \quad \log_r 2 = 0.7 \quad \log_5 = \log \frac{1}{r} = \log 10 - \log r = 1 - 0.7 = 0.3$$

$$(\log \frac{a}{r})n^2 + (\log 9)n - \log 10 = 0 \rightarrow (\log a - \log r)n^2 + (2 \log 3)n - \log 5 - \log r = 0$$

$$0 = \text{معادله} \quad n_1 = 1$$

$$n_2 = \frac{\log 5 + \log r}{\log 3 - \log 5} = \frac{0.3 + 0.7}{0.7 - 0.3} = \frac{1}{0.4} = \frac{11}{4}$$

$$|n_1 - n_2| = |1 - \frac{11}{4}| = \boxed{\frac{13}{4}}$$

$$\log_r 10 = 2.1$$

$$\log_5 2 = 0.5 \rightarrow \log_r 5 = 2$$

$$\log_{10} \frac{1}{r} = \frac{\log \frac{1}{r}}{\log \frac{1}{r}} = \frac{\log 5 \times r}{\log_{10} r} = \frac{\log 5 + \log r}{\log r + \log r} = \frac{2+1}{2.1+1} = \frac{3}{3.1} = \boxed{\frac{15}{19}}$$

$$\log_r a = 1,5 \quad \log_r r = 1,4 \rightarrow \log_r r = 0,4r5$$

$$\log_r^4 = \frac{\log_r^4}{\log_r^4} = \frac{\log_r^{r \times r}}{\log_r^{a \times r}} = \frac{\log_r^r + \log_r^r}{\log_r^a + \log_r^r} = \frac{0,4r5 + 1}{1,5 + 1} = \frac{1,4r5}{r,5} = 0,9r5$$

$$\log_r^n = \log_r^{r^n} = \frac{r}{r} \log_r^r + \log_r^{\frac{r^n}{r}} = n \rightarrow \frac{r}{r} \log_r^r = n - \frac{1}{r} \rightarrow \log_r^r = \frac{r(n-1)}{r}$$

$$\log_r^{r^n} = \log_r^{r \times r} = \frac{\log_r^r}{\frac{1}{r}} + \frac{1}{r} \log_r^r = 1 + \frac{r(n-1)}{r} = \boxed{\frac{r(n+r)}{r}}$$

$$\left(\frac{0,4}{r}\right)^{r(n-1)} = \left(\frac{1,4r5}{r}\right)^{r^n} \rightarrow \left(\frac{r}{a}\right)^{r(n-1)} = \left(\frac{r}{a}\right)^{-r \times r} \rightarrow -r(n-1) = r(n-1)$$

$$\rightarrow r(n-1) = 0 \rightarrow \begin{cases} n = -1 \rightarrow \log_r^{(n+1)} = \log_r^{-1} = \log_r^{-1} \\ n = \frac{1}{r} \rightarrow \log_r^{(n+1)} = \log_r^{\frac{1}{r}} = \log_r^{\frac{r}{r}} = \boxed{\frac{r}{r}} \end{cases}$$

$$\log_r^r = a \rightarrow r^a = r$$

$$\log_r^b = \frac{r}{r} (1+a) \rightarrow \log_r^b = r(1+a) \rightarrow b = r^{r+a} = r \times \left(\frac{r^a}{r}\right)^r = r \times r$$

$$\Rightarrow b = r \times r \rightarrow r \times b - 1 = r \times r - 1 = 100 \rightarrow \boxed{\log_r 100 = r}$$

$$-fa n^r + b n + \frac{1}{r} c = 0$$

$$S: \frac{b}{ra} \rightarrow \frac{1}{\frac{b}{ra}} = \log_r^r \rightarrow b = \frac{ra}{\log_r^r}$$

$$ra = b + c \rightarrow c = ra - b \rightarrow c = ra - \frac{ra}{\log_r^r} = a \left(r - \frac{r}{\log_r^r} \right) \rightarrow \frac{c}{a} = r - \frac{r}{\log_r^r}$$

$$\left(\frac{1}{\log_r^r}\right)^{\frac{c}{a}} \cdot (r)^{\frac{1}{r} \times \frac{c}{a}} = (r)^{\frac{1}{r} \times \left(r - \frac{r}{\log_r^r} \right)} \rightarrow r^{\frac{1}{r} \log_r^r - \frac{1}{r}} = r^{\frac{1}{r} \log_r^r} \times r^{-\frac{1}{r}} = 10^{\frac{1}{r} \log_r^r} \times r^{-\frac{1}{r}}$$

$$\Rightarrow 10^{\frac{1}{r} \log_r^r} \times r^{-\frac{1}{r}} = \left(\frac{1}{r}\right)^{\frac{1}{r}} = \boxed{\sqrt[r]{r}}$$