مهران کاظمی   یازدهم ۲   تکلیف شماره ۴

۱)

$\log_n^m = a \to n^a = m$

$\log_{mn}^{mn^2} = b \Rightarrow \log_{n^{a+1}}^{n^{2a+1}} = b \to \frac{2a+1}{a+1}\log_n^n = b$

$\Rightarrow b = \frac{2a+1}{a+1} \to [b] = \left[\frac{2a+1}{a+1}\right] = \left[\frac{a+a+1}{a+1}\right]$

$= \left[\frac{a}{a+1}+1\right] = \left[\frac{a}{a+1}\right]+1 = \boxed{1}$

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۲) الف) $y = \sqrt{\frac{x}{\log_{\frac{1}{2}}^x}}$ $x>0$

$\frac{x}{\log_{\frac12}^x} \geq 0$ $\log_{\frac12}^x > 0 \to x<1$

$\cap \to 0<x<1 \to (0,1)$

ب) $y = \frac{\log_4^{(x^2-x-2)}}{\sqrt{x^2-1}+1} \to (x-2)(x+1)>0$ $\begin{array}{c} -1 \quad 2 \\ + \,|\, - \,|\, + \end{array}$ $(-\infty,-1)\cup(2,+\infty)$

$\to x^2-1\geq 0$ $(x-1)(x+1)\geq 0$ $\begin{array}{c} -1 \quad 1 \\ + \,|\, - \,|\, + \end{array}$ $(-\infty,-1]\cup[1,+\infty)$

$\xrightarrow{\cap} \boxed{(-\infty,-1)\cup(2,+\infty)}$

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۳)

$2\log_x^a + \log_a^{\sqrt{x}} = 2 \to 2\log_9^a + \log_a^3 = 2$

$\log_3^a + \log_a^3 = 2 \to \underbrace{\log_3^a}_{t} + \frac{1}{\log_3^a} = 2$

$t+\frac{1}{t} = 2 \to t^2-2t+1=0 \to (t-1)^2=0 \to t=1 \to \log_3^a = 1 \to \boxed{a=3}$

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۴)

$\log 2 \simeq 0.3$ $\log 3 \simeq 0.4$

$\left[\log\frac{5}{3}\right]x^2 + (\log 9)x - \log 15 = 0 \to 0.3x^2+0.8x-1.1=0$

$3x^2+8x-11=0 \to \begin{cases} x=-\frac{11}{3} \\ x=1 \end{cases}$

$\log 5 - \log 3 = \log 10 - \log 2 - \log 3 = 1-\underbrace{0.3-0.4}_{0.7} = 0.3$

$1-\left(-\frac{11}{3}\right) = \boxed{\frac{14}{3}}$

$\log 15 = \log 3 + \log\frac{10}{2} = \log 3 + \log 10 - \log 2 = 1.4 - 0.3 = 1.1$

(5

$\log_2 7 = 2.8 \qquad \log_5 2 = 0.5 \rightarrow \dfrac{\log 2}{\log 5} = \dfrac{1}{2} \rightarrow \dfrac{\log 2}{\log \frac{10}{2}} = \dfrac{1}{2} \rightarrow \dfrac{\log 2}{\log 10 - \log 2} = \dfrac{1}{2} \Rightarrow \dfrac{\log 2}{1-\log 2} = \dfrac{1}{2}$

$\log 2 = \dfrac{1}{3}$

$\log_{14} 10$

$= \dfrac{\log 7}{\log 2} = \dfrac{28}{10} \rightarrow \dfrac{\log 7}{\frac{1}{3}} = \dfrac{28}{10} \Rightarrow \log 7 = \dfrac{14}{15}$

$= \dfrac{1}{\log_{10} 14} = \dfrac{1}{\log 2 + \log 7} = \dfrac{1}{\frac{1}{3} + \frac{14}{15}} = \boxed{\dfrac{15}{19}}$

(6

$\log_3 5 = 1.5 \qquad \log_2 3 = 1.6 \rightarrow \log_3 2 = \dfrac{10}{16} = \dfrac{5}{8}$

$\log_{15} 6 = ? \rightarrow \dfrac{\log_3 6}{\log_3 15} = \dfrac{\log_3 2 + \log_3 3}{\log_3 3 + \log_3 5} = \dfrac{\frac{5}{8}+1}{\frac{3}{2}+1} = \boxed{0.65}$

(7

$\log_8 18 = m \qquad \log_4 12 = ?$

$\log_4 12 = \dfrac{\log_8 12}{\log_8 4}$

$\log_8 12 = \log_8 4 + \log_8 3 = \dfrac{2}{3}\log_2 2 + \dfrac{1}{3}\log_2 3 = \dfrac{2}{3} + \dfrac{1}{3}\log_2 3$

$\log_8 4 = \dfrac{2}{3}\log_2 2 = \dfrac{2}{3}$

$\log_8 18 = m \Rightarrow \log_8 9 + \log_8 2 = m \Rightarrow \dfrac{2}{3}\log_2 3 + \dfrac{1}{3}\log_2 2 = m$

$\rightarrow \log_2 3 = \dfrac{3m-1}{2}$

$\dfrac{\frac{2}{3} + \frac{1}{3}\log_2 3}{\frac{2}{3}} = \dfrac{\frac{2}{3} + \frac{1}{3}\left(\frac{3m-1}{2}\right)}{\frac{2}{3}} = \dfrac{2 + \left(\frac{3m-1}{2}\right)}{2} = \dfrac{4+3m-1}{4}$

$= \boxed{\dfrac{3}{4}(m+1)}$

(8

$(0.4)^{2n-1} = \left(\dfrac{125}{8}\right)^{n^2} \Rightarrow \left(\dfrac{2}{5}\right)^{2n-1} = \left(\dfrac{5}{2}\right)^{3n^2} \rightarrow \left(\dfrac{5}{2}\right)^{1-2n} = \left(\dfrac{5}{2}\right)^{3n^2}$

$\log_8 (9n+1) = ?$

$3n^2 = -2n+1 \rightarrow 3n^2 + 2n - 1 = 0$

$n = -1$ غ ق ق

$n = \dfrac{1}{3}$

$\log_8 9\left(\frac{1}{3}\right)+1 = \log_8 4 = \dfrac{2}{3}\log_2 2 = \boxed{\dfrac{2}{3}}$

(9

$\log_{2}^{3} = a$

$\log_{8}^{b} = \frac{3}{2}(1+a)$

$\log(2b-8) = ?$

$\log_{8}^{b} = \frac{3}{2}(1+\log_{2}^{3}) = \frac{3}{2} + \frac{3}{2}\log_{2}^{3}$

$\Rightarrow \log_{8}^{b} - \log_{8}^{9} = \frac{3}{2} \rightarrow \log_{8}^{\frac{b}{9}} = \frac{3}{2}$

$8^{\frac{3}{2}} = \frac{b}{9} \Rightarrow b = 36$

$\sqrt[32]{x^{2}\,24} = 4$

$\log(108-8) = \log 100 = \boxed{2}$

(10

$-4ax^2 + bx + \frac{1}{2}c = 0 \rightarrow \frac{1}{\frac{-b}{-4a}} = \log 4 \Rightarrow \frac{4a}{b} = \log 4$

$\frac{a}{b} = \frac{\log 4}{4} = \frac{2\log 2}{4} = \frac{\log 2}{2}$

$\frac{b+c}{2} = a \rightarrow \cancel{\dots} \; c = 2a - b \xrightarrow{\div a} \frac{c}{a} = 2 - \frac{b}{a} = 2 - \left(\frac{2}{\log 2}\right) = 2\left(1 - \frac{1}{\log 2}\right)$

$= 2(\underbrace{1 - \log_{2}^{10}}_{\log_{2}^{2} - \log_{2}^{10}}) = \cancel{\dots} \; 2(\log_{2}^{0.2})$

$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}} = \left(2^{-\frac{1}{8}}\right)^{\frac{c}{a}} = 2^{-\frac{1}{4}\log_{2}^{0.2}} = 2^{\log_{16}^{5}} \Rightarrow 5^{\log_{16}^{2}} = 5^{\frac{1}{4}\log_{2}^{2}} = \boxed{\sqrt[4]{5}}$