عسل کاظمی — یازدهم تجربی گروه ج سپیده‌ها

سوال ۱ ←

$\log_n m = a \to n^a = m \qquad \log_{mn} m^2 n \to \log_{n^{a+1}} n^{2a+1}$

$\to b = \frac{2a+1}{a+1} \qquad \log_n n = b$

$b = \frac{2a+1}{a+1} \to [b] = \left[\frac{2a+1}{a+1}\right] \to [b] = \left[1 + \frac{a}{a+1}\right]$

$\to [b] = \left[\frac{a}{a+1}\right] + 1 \to b = 1 + 0 = \boxed{1}$

سوال ۲ ←

الف) $y = \sqrt{\frac{x}{\log_{\frac{1}{2}} x}}$

$\log_{\frac{1}{2}} x \to x > 0$

$x \geq 0$ به دلیل رادیکال (ارتباط؟)

$\Longrightarrow 0 < x < 1 \to$ بین این بازه ما عدد صحیح نداریم

ب) $\frac{\log_4 (x^2 - x - 2)}{\sqrt{x^2-1} + 1} \to \sqrt{x^2-1} + 1 = 0 \to \sqrt{x^2-1} = -1$

هیچ وقت اتفاق نمی‌افتد

$\log_4 (x^2 - x - 2) \to x^2 - x - 2 > 0 \to (-\infty, -1) \cup (2, +\infty)$ ①

$x^2 - 1 \geq 0 \to x^2 \geq 1 \to (-\infty, -1] \cup [+1, +\infty)$ ②

$\to$ ① ② $\xrightarrow{\cap} (-\infty, -1) \cup (2, +\infty)$

سوال ۳ ←

$2\log_x 9 + \log_a \sqrt{x} = 2 \to 2\log_{9} a + \log_a 3 = 2 \quad (9 = 3^2)$

[crossed out] $\to \log_3 a + \log_a 3 = 2$

$\to \log_3 a + \frac{1}{\log_3 a} = 2 \to \log_3 a = \log_a 3 = 1 \to \boxed{a = 3}$

سوال ۴ ←

$(\log\frac{5}{3})x^2+(\log 9)x-\log 15=0$

$(\log 5-\log 3)x^2+2(\log 3)x-\log 5-\log 3=0$

$(\log 5)x^2-0.4x^2+0.8x-\log 5-0.4=0$

$\to(\log\frac{10}{2})x^2-0.4x^2+0.8x-\log(\frac{10}{2})-0.4=0$

$\to(\log 10-\log 2)x^2-0.4x^2+0.8x-(\log 10-\log 2)-0.4=0$

$(1-0.3)x^2-0.4x^2+0.8x-0.7-0.4=0$

$\to 0.3x^2+0.8x-1.1 \Rightarrow x=-\frac{11}{3},\ x=1 \to 1-(-\frac{11}{3})=\frac{14}{3}$

سوال ۵ ←

$\log_2 7=2.8 \qquad \log_5 2=0.5 \qquad \log_{14} 10=?$

$\frac{\log 2}{\log 5}=\frac{1}{2}\to\frac{\log 2}{\log\frac{10}{2}}=\frac{1}{2}\to\frac{\log 2}{\log 10-\log 2}=\frac{1}{2}$

$\to\frac{\log 2}{1-\log 2}=\frac{1}{2}\to 2\log 2=1-\log 2\to 3\log 2=1\to\log 2=\frac{1}{3}$

$\frac{\log 7}{\log 2}=\frac{14}{5}\to\frac{\log 7}{\frac{1}{3}}\to\log 7=\frac{14}{15}$

$\frac{1}{\log\frac{14}{10}}=\frac{1}{\log\frac{2\times 7}{10}}\to\log 2+\log 7\to\frac{1}{\frac{1}{3}+\frac{14}{15}}=\frac{19}{15}$

سوال ۶ ←

$\log_3 5=1.5 \qquad \log_2 3=1.6 \qquad \log_{15} 6=?$

$\log_3 5\times\log_2 3=\log_2 5\to\log_3 5\times\log_2 3\to 1.5\times 1.6=\log_2 5$

$\to\log_2 5=2.4\to\log_{15} 6=\frac{\log_2 6}{\log_2 15}\to\log_{15} 6\to\frac{\log_2 2+\log_2 3}{\log_2 3+\log_2 5}$

$\to\frac{1+1.6}{1.6+2.4}=\frac{2.6}{4}=0.65$

سوال ۷ ←

$\log_8 18 = m$ $\qquad \log_4 12 = ?$ $\qquad \log_4 12 \to \frac{\log_8 12}{\log_8 4}$

$\to \log_8 12 = \log_8 4 + \log_8 3 = \log_{2^3} 2^2 + \log_{2^3} 3$

$\to \frac{2}{3}\log_2 2 + \frac{1}{3}\log_2 3 \to \frac{2}{3} + \frac{1}{3}\log_2 3$

$\to \log_8 4 = \log_{2^3} 2^2 = \frac{2}{3}\log_2 2 = \frac{2}{3}$

$\log_8 18 = \log_8 2 + \log_8 9 \to \log_{2^3} 2 + \log_{2^3} 3^2 \to \frac{1}{3}\log_2 2 + \frac{2}{3}\log_2 3 = m$

$\to \frac{1}{3} + \frac{2}{3}\log_2 3 = m \to \log_2 3 = \frac{3m-1}{2}$

$\to \log_4 12 = \frac{\frac{2}{3} + \frac{1}{3}\log_2 3}{\frac{2}{3}} = \frac{\frac{2}{3} + \frac{1}{3}\left(\frac{3m-1}{2}\right)}{\frac{2}{3}} = \frac{2 + \left(\frac{3m-1}{2}\right)}{2}$

$\to \frac{3+3m}{2} \to \boxed{\frac{3}{2}(m+1)}$

سوال ۸ ←

$(0.4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \to \left(\frac{2}{5}\right)^{2x-1} = \left(\frac{5}{2}\right)^{3x^2}$

$\to 3x^2 = 1-2x \to 3x^2+2x-1 \to x=-1$ ✗ $\quad x = \frac{1}{3}$ ✓

$\to \log_8 \left(9\left(\frac{1}{3}\right)+1\right) = \log_8 4 \to \log_{2^3} 2^2 \to \frac{2}{3}\log_2 2 \to \boxed{\frac{2}{3}}$

سوال ۹ ←

$\log_2 3 = a \qquad \log_8 b = \frac{2}{3}(1+a)$

$\log_8 b = \frac{2}{3}(1+\log_2 3) \Rightarrow \log_8 b = \frac{2}{3} + \frac{2}{3}\log_2 3 \to \log_8 b = \frac{2}{3} + \log_8 9$

$\to \log_8 b - \log_8 9 \to b = 36 \to \log(108-8)$

$\to \log 100 = \boxed{2}$

سوال ۱۰ ←

$-4x^2 + bx + \frac{1}{2}c = 0 \qquad \frac{1}{\frac{-b}{-4a}} \to \log_4 \to \log_4 = \frac{4a}{b}$

$$\frac{a}{b}=\frac{\log 4}{4} \rightarrow \frac{2\log 2}{4}=\frac{\log 2}{2} \, *$$

$$* \xrightarrow{\text{ضرب}} a \quad 2\left(\frac{\log 2}{2}\right)=1+\frac{c}{b} \rightarrow \frac{c}{b}=\log 2-1$$

(1) $\frac{c}{b}=\log 2-1$

(2) $\frac{a}{b}=\frac{\log 2}{2} \rightarrow \frac{c}{a}=\frac{\log 2-1}{\frac{\log 2}{2}} \rightarrow \frac{\log 2-\log 10}{\frac{\log 2}{2}} \rightarrow \frac{\log \frac{1}{5}}{\frac{\log 2}{2}}$

$$\rightarrow \frac{\log \frac{1}{5}}{\frac{1}{2}\log 2}=\log_{\sqrt{2}} \frac{1}{5} \rightarrow \left(\frac{1}{\sqrt{2}}\right)^{\frac{c}{a}}=\left(\frac{1}{\sqrt{2}}\right)^{\log_{\sqrt{2}} \frac{1}{5}}$$

$$\rightarrow \left(\frac{1}{5}\right)^{\log_{\sqrt{2}} \frac{1}{2\sqrt{2}}} \rightarrow \left(\frac{1}{5}\right)^{\log_{2^{\frac{1}{2}}} 2^{-\frac{1}{8}}} \rightarrow \left(\frac{1}{5}\right)^{-\frac{1}{4}}$$

$$\rightarrow (5)^{\frac{1}{4}} \rightarrow \boxed{\sqrt[4]{5}}$$