دریا دریانی دوردستم

(۱)

$n^a = m$

$\log_{mn}^{m^2 n} = b \Rightarrow \log_{n^a \times n}^{n^{2a} \times n} = b \Rightarrow \log_{n^{a+1}}^{n^{2a+1}} = b \to \frac{2a+1}{a+1} \log_n^n = b \Rightarrow \left[\frac{a+a+1}{a+1}\right] = [b]$

$\left[1+\frac{a}{a+1}\right] = [b] \Rightarrow 1+\left[\frac{a}{a+1}\right] = [b] \Rightarrow 0 < \frac{a}{a+1} < 1 \to [b] = 1$

(۲)

الف) $y = \sqrt{\frac{x}{\log_{\frac{1}{2}}^{x}}}$

$x > 0$

$\log_{\frac{1}{2}}^{x} \neq 0 \to x \neq 1$

$\begin{array}{c|c|c} & 0 & 1 \\ \hline & + & - \end{array} \to D_f = (0,1)$

ب) $y = \frac{\log_4^{(x^2-x-2)}}{\sqrt{x^2-1}+1}$

$\sqrt{x^2-1} \neq - \to \checkmark$ همواره برقرار

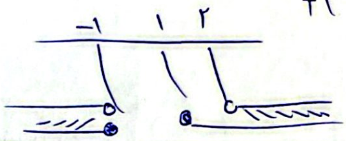
$(x-1)(x+1) \geq 0$

$\begin{array}{c|c|c} -1 & & +1 \\ \hline & - & + \end{array}$

$x^2-x-2 > 0$

$(x+1)(x-2)$

$\begin{array}{c|c|c|c} & -1 & & 2 \\ \hline + & & - & + \end{array}$



$D_f = (-\infty,-1) \cup (2,+\infty)$

(۳)

$2\log_x^a + \log_a^{x^{\frac{1}{2}}} \Rightarrow 2\log_x^a + \frac{1}{2}\log_a^x \Rightarrow 2\log_x^a + 2\log_x^a = 2$

$\log_x^a = \frac{2}{4} \to a = x^{\frac{1}{2}} \quad x = 9 \quad a = 9^{\frac{1}{2}} \to \sqrt{9} \Rightarrow \boxed{a = 3}$

(۴)

$(\log 5 - \log 3)x^2 + (2\log 3)x - (\log 3 + \log 5) = 0$

$\frac{c}{a} = \frac{-\log 3 - \log 5}{\log 5 - \log 3} = \frac{-0.4 - \log 10 + \log 2}{\log 10 - \log 2 - 0.4} = \frac{-0.4 - 1 + 0.3}{1 - 0.3 - 0.4} = \frac{-1.1}{0.3} = -\frac{11}{3}$

$x_1 - x_2 = 1 - \left(-\frac{11}{3}\right) = \boxed{\frac{14}{3}}$

(۵)

$\log_5^2 \times \log_2^7 = 0.5 \times 2.8 \Rightarrow \log_5^7 = 1.4 \qquad \log_{14}^{10} = \frac{\log_5^{10}}{\log_5^{14}}$

$\log_{14}^{10} = \frac{\log_5^2 + \log_5^5}{\log_5^2 + \log_5^7} = \frac{0.5+1}{0.5+1.4} = \frac{1.5}{1.9} = \boxed{\frac{15}{19}}$

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$$\log_3^5 \times \log_2^3 = \log_2^5 = 2.4$$

$$\log_{15}^6 = \frac{\log_2^6}{\log_2^{15}} = \frac{\log_2^2 + \log_2^3}{\log_2^3 + \log_2^5} = \frac{1 + 1.6}{1.6 + 2.4} = \frac{2.6}{4} = \frac{1.3}{2} = 0.65$$

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$$\log_4^{12} = \frac{\log_8^{12}}{\log_8^4} \Rightarrow \log_8^{12} = \log_8^4 + \log_8^3 \Rightarrow \frac{2}{3} + \frac{1}{3}\log_2^3$$

$$\log_8^4 = \frac{2}{3}\log_2^2 = \frac{2}{3}$$

$$\log_8^{18} = m \to \log_8^9 + \log_8^2 = m \to \frac{2}{3}\log_2^3 + \frac{1}{3}\log_2^2 = m \to \frac{2}{3}\log_2^3 + \frac{1}{3} = m$$

$$\log_2^3 = \frac{3m-1}{2} \qquad \log_4^{12} = \frac{\frac{2}{3} + \frac{1}{3}\log_2^3}{\frac{2}{3}} = \frac{\frac{2}{3} + \frac{1}{3}\left(\frac{3m-1}{2}\right)}{\frac{2}{3}} = \frac{4+3m-1}{4} = \boxed{\frac{3}{4}(m+1)}$$

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$$\left(\frac{4}{10}\right)^{2x-1} = \left(\frac{5^3}{2^3}\right)^{x^2} \Rightarrow \left(\frac{2}{5}\right)^{2x-1} = \left(\frac{5}{2}\right)^{3x^2} \Rightarrow 2x-1 = -3x^2$$

$$3x^2 + 2x - 1 = 0 \Rightarrow x^2 + 2x - 3 = 0 \longrightarrow \left(x + \frac{3}{2}\right)\left(x - \frac{1}{3}\right) = 0$$

$x = -1 \Rightarrow$ غیرقابل قبول $\to 9x + 1 \Rightarrow -9 + 1 = -8$

$$x = \frac{1}{3} \checkmark \to \log_{2^3}^{(\cancel{9x}+1)} = \log_{2^3}^{2^2} = \frac{2}{3}\log_2^2 = \boxed{\frac{2}{3}}$$

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$$\frac{1}{3}\log_2^b = \frac{2}{3}(a+1) \to \log_2^b = 2a+2$$

$$2^{2a+2} = b \to (2^a)^2 \times 4 = 36 \to b = 36$$

$$\log(3b - 8) \Rightarrow \log 100 = 2$$

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$$S = \frac{-b}{-4a} \Rightarrow \frac{b}{4a} = \log 4 \xrightarrow{} \underset{①}{\frac{a}{b}} = \frac{\log 4}{4} = \frac{2\log 2}{4} = \frac{\log 2}{2}$$

$2a = b + c \xrightarrow{\div b} \frac{2a}{b} = 1 + \frac{c}{b} \to$ با توجه به ① ، $\frac{a}{b} = \frac{\log 2}{2} \to 2\left(\frac{\log 2}{2}\right) = 1 + \frac{c}{b}$

$$\underset{②}{\frac{c}{b}} = -1 + \log 2 \xrightarrow{①,②} \frac{c}{a} = \frac{-1 + \log 2}{\frac{\log 2}{2}} \Rightarrow \frac{c}{a} = \frac{\log 2 - \log 10}{\frac{1}{2}\log 2} = \frac{\log\frac{2}{10}}{\frac{1}{2}\log 2} = \frac{\log\frac{1}{5}}{\frac{1}{2}\log 2}$$

$$\frac{\log\frac{1}{5}}{\log\sqrt{2}} = \log_{\sqrt{2}}^{\frac{1}{5}} \longrightarrow \left(\frac{1}{\sqrt{2}}\right)^{4a} \Rightarrow \left(\frac{1}{5}\right)^{\log_{\sqrt{2}}^{\frac{1}{\sqrt[4]{2}}}} = \left(\frac{1}{5}\right)^{\log_{\frac{1}{2}}^{2^{-\frac{1}{8}}}} = \frac{1}{5}\left(\frac{-\frac{1}{8}}{-\frac{1}{2}}\right)\log_2^2 = \left(\frac{1}{5}\right)^{-\frac{1}{4}} = \sqrt[4]{5}$$

جواب $= \boxed{\sqrt[4]{5}}$