

(۱)

$$n^a = m$$

$$\log_{mn}^{m^n} = b \Rightarrow \log_{n^a}^{n^{ra} \times n} = b \Rightarrow \log_{n^{a+1}}^{n^{ra+1}} = b \rightarrow \frac{ra+1}{a+1} \log_n^n = b \Rightarrow \left[\frac{a+a+1}{a+1} \right] = [b]$$

$$\left[1 + \frac{a}{a+1} \right] = [b] \Rightarrow 1 + \left[\frac{a}{a+1} \right] = [b] \Rightarrow 0 < \frac{a}{a+1} < 1 \rightarrow [b] = 1$$

(۲)

الف) $y = \sqrt{\frac{x}{\log_{\frac{x}{p}} x}}$

ب) $y = \frac{\log_r^{(r^r - r - r)}}{\sqrt{x^r - 1} + 1}$

الف) $\log_{\frac{x}{p}} x \neq 0 \rightarrow x \neq 1$

ب) $\sqrt{x^r - 1} \neq -1 \rightarrow$ ✓

$(x-1)(x+1) \geq 0$

$\frac{-1}{x-1} \leq \frac{-1}{x+1}$

$D_f = (-\infty, -1) \cup (1, +\infty)$

(۳)

$$r \log_x^a + \log_a^{x^{\frac{1}{r}}} \Rightarrow r \log_x^a + \frac{1}{r} \log_a^x \Rightarrow r \log_x^a + r \log_x^a = r$$

$$\log_x^a = \frac{r}{x} \rightarrow a = x^{\frac{1}{r}} \quad n=9 \quad a = 9^{\frac{1}{3}} \rightarrow \sqrt[3]{9} \Rightarrow a = \sqrt[3]{9}$$

(۴)

$$(\log_{\Delta} - \log_r) x^r + (r \log_r) x - (\log_r + \log_{\Delta}) = 0$$

$$\frac{c}{a} = \frac{-\log_r - \log_{\Delta}}{\log_{\Delta} - \log_r} = \frac{-r - \log 1 + \log r}{\log 1 - \log r - r} = \frac{-r - 1 + r^r}{1 - r^r - r} = \frac{-1}{r} = -\frac{1}{r}$$

$$x_1 - x_2 = 1 - \left(-\frac{1}{r}\right) = \frac{1+r}{r}$$

(۵)

$$\log_{\Delta}^r \times \log_r^{\Delta} = \Delta \times r, 1 \Rightarrow \log_{\Delta}^{\Delta} = 1, \Delta$$

$$\log_{\frac{1}{\Delta}}^{\Delta} = \frac{\log_{\Delta}^{\Delta}}{\log_{\frac{1}{\Delta}}^{\Delta}}$$

$$\log_{\frac{1}{\Delta}}^{\Delta} = \frac{\log_{\Delta}^{\Delta} + \log_{\Delta}^{\Delta}}{\log_{\Delta}^{\Delta} + \log_{\Delta}^{\Delta}} = \frac{\Delta + 1}{\Delta + 1, \Delta} = \frac{1, \Delta}{1, \Delta} = \left[\frac{1, \Delta}{1, \Delta} \right]$$

