یازدهم دختر C — تکلیف ۲۳ — نیوشا دهقان سمیع‌پور

(۱)

$\log_{mn}^{m^2n}=b \to \log_{mn}^{mn}+\log_{mn}^{m}=b \to \log_{mn}^{m}=b-1 \to \log_{n^a\cdot n}^{n^a}=b-1$

$\log_{n^{a+1}}^{n^a}=b-1 \to b-1=\frac{a}{a+1} \longrightarrow a>0 \to a+1>a \quad \frac{a}{a+1}>0 \to b=1/m \to [b]=1$

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(۲)

الف) $y=\sqrt{\frac{x}{\log_{\frac{1}{2}}^{x}}} \quad \frac{x}{\log_{\frac{1}{2}}^{x}} \to \log_{\frac{1}{2}}^{x}\neq 0 \to x\neq 1,\ x>0 \to D_f=(0,+\infty)-\{1\}$

ب) $y=\frac{\log_{4}^{(x^2-x-2)}}{\sqrt{x^2-1}+1} \longrightarrow x^2-1\geq 0 \to x^2\geq 1 \to x\geq 1$ یا $x\leq -1$ $\boxed{1}$

$x^2-x-2>0$

$(x-2)(x+1)>0 \to$ $\begin{array}{ccccc} & -1 & & 2 & \\ + & & - & & + \end{array}$ $\to x\to(-\infty,-1)\cup(2,+\infty)$ $\boxed{2}$

هیچ‌وقت صفر نمی‌شود

$\boxed{1}\cap\boxed{2} \to (-\infty,-1)=D_f$

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(۳)

$x=9 \leftarrow 2\log_x^a+\log_a^{\sqrt{x}}=2$

$2\log_{9}^{a}+\log_{a}^{3}=2$ ($9=3^2$)

$\frac{2}{2}\log_3^a+\log_a^3=2 \to \frac{1}{\log_a^3}+\log_a^3=2 \longrightarrow \log_a^3=1 \to \boxed{a=3}$

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(۴)

$[\log\frac{5}{2}]x^2+(\log 9)x-\log 15=0 \longleftarrow \log 3\approx 0/4\ ,\ \log 2\approx 0/3$

$\log\frac{5}{2}=\log_{10}^{5}-\log_{10}^{3}=(\log_{10}^{10}-\log_{10}^{2})-\log_{10}^{2}=0/4$ $\quad \log 9=\log_{10}^{3^2}=2\log^3=0/8$

$\log 15=\log^{\frac{30}{2}}=\log_{10}^{10}+\log_{10}^{3}-\log_{10}^{2}=1/1$ $\quad \frac{3}{10}x^2+\frac{8}{10}x-\frac{11}{10}=0 \longrightarrow 3x^2+8x-11=0 \to x^2+8x-33=0$

$1+\frac{11}{3}=\boxed{\frac{14}{3}}$ یا $-\frac{11}{3}-\frac{3}{3}=\boxed{-\frac{14}{3}}$ $\quad \boxed{x=-\frac{11}{3}}$ و $\boxed{x=1} \leftarrow (x+11)(x-3)$

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(۵)

$\log_2^7=2/8 \quad \log_5^2=0/5 \longrightarrow \log_2^5=2 \quad \log_{14}^{10}=\frac{\log_2^{10}}{\log_2^{14}}=\frac{\log_2^{5\times 2}}{\log_2^{7\times 2}}=\frac{\log_2^5+\log_2^2}{\log_2^7+\log_2^2}=\frac{3}{3/8}=\frac{30}{38}$

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(۶)

$\log_3^5=1/5 \longrightarrow \log_2^3=1/6 \longrightarrow \log_3^2=\frac{1}{\frac{16}{10}}=\frac{10}{16}=\frac{5}{8}$

$\log_{15}^{6}=\frac{\log_3^6}{\log_3^{15}}=\frac{\log_3^2+\cancel{\log_3^3}\ 1}{\log_3^5+\cancel{\log_3^3}\ 1}=\frac{\frac{13}{8}}{\frac{25}{10}}=\frac{13}{20}$

(7)

$\log_8^{18} = x \to \log_8^{9} + \log_8^{2} = \frac{2}{3}\log_2^{3} + \frac{1}{3} = m \to \frac{2}{3}\log_2^{3} = m - \frac{1}{3}$

$\log_4^{12} = \log_4^{3} + \cancel{\log_4^{4}}_{1} = \frac{1}{2}\log_2^{3} + 1 \to \frac{1}{2} \times \frac{3}{2} \times (m - \frac{1}{3}) + 1 \qquad \log_2^{3} = (m - \frac{1}{3})\frac{3}{2}$

$= \frac{3}{4}m - \frac{1}{4} + 1 = \frac{3}{4}m + \frac{3}{4}$

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(8)

$(0.4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \to \left(\frac{2}{5}\right)^{2x-1} = \left(\frac{5^3}{2^3}\right)^{x^2} \to \left(\frac{5}{2}\right)^{1-2x} = \left(\frac{5}{2}\right)^{3x^2}$

$3x^2 = 1 - 2x \to 3x^2 + 2x - 1 = 0 \to x^2 + 2x - 3 \to x = \frac{1}{3}$ ق ق

$x = -1 \to$ غ ق ق

$\log_8^{(9x+1)} = \log_8^{9 \times \frac{1}{3} + 1} = \log_8^{3+1} = \log_{2^3}^{2^2} = \boxed{\frac{2}{3}}$

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(9)

$\log_2^{3} = a \qquad \log_8^{b} = \frac{2}{3}(1+a) \to \frac{2}{3}(1 + \log_2^{3}) = \frac{2}{3}(\log_2^{2} + \log_2^{3})$

$\frac{2}{3}(\log_2^{6}) = \frac{2}{3}\log_2^{6} = \log_8^{36} = \log_8^{b} \to b = 36 \qquad \log^{3b-8} = \log_{10}^{100} = 2$

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(10)

$-4ax^2 + bx + \frac{1}{2}c = 0$ [illegible] $\to \frac{-b}{-4a} = \frac{b}{4a} \longrightarrow \frac{4a}{b} \qquad \frac{4a}{b} = \log_{10}^{4}$

$\frac{c}{2a} = 1 - \log_2^{10} \leftarrow \log_2^{10} = 1 - \frac{c}{2a} \leftarrow \frac{1}{\log_{10}^{2}} = \frac{2a-c}{2a} \qquad \frac{4a}{b} = 2\log_{10}^{2} \to \log_{10}^{2} = \frac{2a}{b} \to \log_2^{10} = \frac{2a}{2a-}$

$\log_2^{2} - \log_2^{10} = \log_2^{\frac{1}{5}} \to \log_2^{\frac{1}{5}} = \frac{c}{2a} \to \frac{c}{a} = 2\log_2^{\frac{1}{5}} \to \log_2^{\frac{1}{25}}$

$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}} = \left(\frac{1}{\sqrt[8]{2}}\right)^{\log_2^{\frac{1}{25}}} \to \left(\frac{1}{25}\right)^{\log_2^{2^{-\frac{1}{8}}}} = \left(\frac{1}{25}\right)^{-\frac{1}{8}} = \sqrt[8]{25} = \sqrt[4]{5}$