تکلیف شماره ۲۳ دینا جمشیدی یازدهم تجربی

① $\log_n m = a \qquad \log_{mn} m^2 n = b$

$a > 0 \longrightarrow [b] = ?$

$m = n^a$

$\log_{mn} m^2 n = \log_{n^a \times n} (n^a)^2 \times n =$

$\log_{n^{a+1}} n^{2a+1} = \frac{2a+1}{a+1} \log_n n =$

$\frac{2a+1}{a+1} = \frac{a+1+a}{a+1} = 1 + \frac{a}{a+1} \longrightarrow [\ ]$

$1 + \left[\frac{a}{a+1}\right] = 1$

② الف) $y = \sqrt{\frac{x}{\log_{\frac{1}{2}} x}}$ $\quad x \longrightarrow 0 =$ ریشه $\quad \log_{\frac{1}{2}} x \longrightarrow 1 =$ ریشه $\quad x > 0$

$x > 1 \rightarrow \log_{\frac{1}{2}} x < 0 \qquad x < 1 \rightarrow \log_{\frac{1}{2}} x > 0$

	0		1	
−		+		−

$\longrightarrow D_f = (0, 1)$

ب) $\frac{\log_2 (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$

$x^2 - x - 2 > 0 \qquad (x+1)(x-2) > 0$

$\rightarrow (-\infty, -1) \cup (2, +\infty)$

$\sqrt{x^2 - 1} \neq -1$ همواره برقرار ✓

$D_f = (-\infty, -1) \cup (2, +\infty)$

③ $2\log_x a + \log_a \sqrt{x} = 2 \qquad x = 9$

$a = ?$

$x = 9 \rightarrow 2\overbrace{\log_9 a}^{\frac{1}{2}\log_3 a} + \log_a 3 = 2$

$\log_3 a + \log_a 3 = 2 \qquad \log_3 a = t$

$t + \frac{1}{t} = 2 \qquad \frac{t^2+1}{t} = 2$

$t^2 - 2t + 1 = 0 \qquad (t-1)^2 = 0 \qquad t = 1$

$\log_3 a = 1 \qquad a = 3$

s.a.m

(6)

$\log_3 5 = 1.5$

$\log_2 3 = 1.6$

$\log_5 6 = ?$

$\dfrac{\log 5}{\log 3} = \dfrac{3}{2}$ $\qquad \dfrac{\log 10 - \log 2}{\log 3} = \dfrac{3}{2}$

$2 - 2\log 2 = 3\log 3$ $\quad 2 - 2\log 2 = 4.8 \log 2$

$\log_2 3 = \dfrac{\log 3}{\log 2} = 1.6$ $\quad \log 2 = \dfrac{1}{3.4}$

$\log 3 = \dfrac{1.6}{3.4}$ $\qquad \log_5 6 = \dfrac{\log 6}{\log 5}$

$\dfrac{\log 2 + \log 3}{\underbrace{\log 3 + \log 5}} = \dfrac{\dfrac{1.6+1}{3.4}}{\dfrac{0.6+3.4}{3.4}} = 0.65$

$\log 10 - \log 2 = 1 - \log 2$

$1.6 \log 2 + 1 - \log 2 = 0.6 \log 2 + 1$

(4)

$\log 2 \simeq 0.3$ $\qquad (\log_3 5)x^2 + (\log 9)x$

$\log 3 \simeq 0.4$ $\qquad - \log 15 = 0$

اختلاف ریشه ها؟

$\log_3 5 = \log 5 - \log 3$

$\log 5 = ?$ $\quad \log \dfrac{10}{2} = \underbrace{\log 10}_{1} - \underbrace{\log 2}_{0.3} = 0.7$

$0.7 - 0.4 = \boxed{0.3}$

$\log 9 = \log 3^2 = 2 \cdot \log 3 = \boxed{0.8}$

$\log 15 = \log 3 \times 5 = \log 3 + \log 5 =$

$0.4 + 0.7 = 1.1$

فاصله $= \dfrac{\sqrt{\Delta}}{|a|}$ $\qquad \Delta = 0.64 - \overbrace{4(0.3)(-1.1)}^{-1.32}$

$\Delta = 1.96$

$\dfrac{\sqrt{1.96}}{0.3} = \dfrac{1.4}{0.3} = \dfrac{14}{3}$

(7)

$\log_8 18 = m$ $\qquad \log_8 12 = \log_{2^3} 3 + \log_{2^3} 2^2$

$\log_4 12 = ?$ $\qquad \dfrac{2}{3}\cancel{\log_2 2} + \dfrac{1}{3}\log_2 3$

$\log_{2^3} 3^2 + \log_{2^3} 2 = \dfrac{2}{3}\log_2 3 + \dfrac{1}{3}\cancel{\log_2 2} = m$

$\dfrac{2}{3}\log_2 3 = m - \dfrac{1}{3}$ $\qquad \log_2 3 = \dfrac{3m-1}{2}$

$\log_8 12 = \dfrac{2}{3} + \dfrac{3m-1}{6}$ $\qquad \dfrac{3}{4}(m+1)$

$\log_8 4 = \log_{2^3} 2^2 = \dfrac{2}{3}\cancel{\log_2 2} = \dfrac{2}{3}$

جواب $= \log_4 12 = \dfrac{\log_8 12}{\log_8 4} = \dfrac{\dfrac{2}{3} + \dfrac{3m-1}{6}}{2/3} =$

(5)

$\log_2 7 = 2.8$ $\qquad \log_5 2 = \dfrac{1}{2}$

$\log_5 2 = 0.5$ $\qquad \dfrac{\log 2}{\underbrace{\log 5}_{\log \frac{10}{2}}} = \dfrac{1}{2}$ $\qquad \dfrac{\log 2}{\cancel{\log 10} - \log 2} = \dfrac{1}{2}$

$\log_{14} 10 = ?$

$2\log 2 = 1 - \log 2 \longrightarrow \log 2 = \dfrac{1}{3}$

$\log_2 7 = \dfrac{\log 7}{\log 2} = 2.8 \rightarrow \log 7 = \dfrac{1}{3} \times 2.8 = \dfrac{14}{15}$

$\log_{14} 10 = \dfrac{\cancel{\log 10}}{\log 14} = \dfrac{1}{\log 2 + \log 7} = \dfrac{1}{\dfrac{1}{3} + \dfrac{14}{15}} = \dfrac{15}{19}$

s.a.m

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$(۰/۴)^{۲x-۱} = \left(\frac{۱۲۵}{۸}\right)^{x^۲}$

$\log_{۸}^{۹x+۱} = ?$

$\log_{۰/۴}\left(\frac{۱۲۵}{۸}\right)^{x^۲} = ۲x-۱$

$x^۲\left(\underbrace{\log_{۰/۴}\frac{۱۲۵}{۸}}\right) = ۲x-۱$

$\log \frac{۱۲۵}{۸} \to \log ۱۲۵ - \log ۸ = \log ۵^۳ - \log ۲^۳$

$\log \frac{۲}{۵} = \quad = ۳(\log ۵ - \log ۲)$

$\to \log ۲ - \log ۵$

$\frac{۳(\log ۵ - \log ۲)}{(\log ۲ - \log ۵)} = -۳$

$\to -۳x^۲ = ۲x-۱ \qquad ۳x^۲+۲x-۱=۰ \begin{cases} -۱ \\ \frac{۱}{۳} \end{cases}$

$a+c=b$

$\log_۸^{۹x+۱} = \begin{cases} x=-۱ \to \log_۸^{-۸} & \text{غ ق ق} \\ x=\frac{۱}{۳} \to \log_۸^۴ = \log_{۲^۳}^{۲^۲} = \end{cases}$

$\frac{۲}{۳}\cancel{\log_۲^۲} = \frac{۲}{۳}$

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$\log_۲^۳ = a$

$\log_۸^b = \frac{۲}{۳}(۱+a)$

$\log(۳b-۸) = ?$

$\to \log_۸^b = \frac{۲}{۳} + \underbrace{\frac{۲}{۳}a} = \log_{۲^۳}^{۳^۲} = \log_۸^۹$

$\log_۸^b = \frac{۲}{۳} + \log_۸^۹$

$\to \frac{۲}{۳} = \log_۸^۴ = \log_{۲^۳}^{۲^۲} = \frac{۲}{۳}$

$\log_۸^b = \log_۸^۴ + \log_۸^۹ = \log_۸^{۳۶}$

$b = ۳۶ \qquad \log_{۱۰}^{(۱۰۰)} = ۲$

۱۰

$-۴ax^۲ + bx + \frac{۱}{۲}c = ۰$

$\frac{۴a}{b} = \log ۴ \qquad \frac{b+c}{۲} = a$

$\frac{a}{b} = \frac{\log ۴}{۴} = \frac{۲\log ۲}{۴} = \frac{\log ۲}{۲}$

$۲a = b+c \xrightarrow{\div b} \frac{۲a}{b} = ۱ + \frac{c}{b}$

$\log ۲ = ۱ + \frac{c}{b}$

$\frac{c}{b} = \log ۲ - ۱$

s.a.m

ادامه سوال ۱۵ :

$$\begin{cases} \dfrac{c}{b} = -1 + \log 2 \\[2mm] \dfrac{a}{b} = \dfrac{\log 2}{2} \end{cases} \qquad \frac{c}{a} = \frac{\log 2 - 1}{\dfrac{\log 2}{2}}$$

$$\frac{c}{a} = \frac{\log 2 - \log 10}{\frac{1}{2}\log 2} \quad \log \frac{2}{10} = \log \frac{1}{5} \qquad \log 2^{\frac{1}{2}} = \log \sqrt{2}$$

$$\frac{c}{a} = \frac{\log \frac{1}{5}}{\log \sqrt{2}} = \log_{\sqrt{2}} \frac{1}{5}$$

$$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}} = \left(\frac{1}{\sqrt[8]{2}}\right)^{\log_{\sqrt{2}} \frac{1}{5}}$$

$$\left(\frac{1}{5}\right)^{\log_{\sqrt{2}} \frac{1}{\sqrt[8]{2}}} \longrightarrow \log_{2^{\frac{1}{2}}} 2^{-\frac{1}{8}} = \frac{-\frac{1}{8}}{\frac{1}{2}} = -\frac{1}{4}$$

$$= \left(\frac{1}{5}\right)^{-\frac{1}{4}} = 5^{\frac{1}{4}} = \sqrt[4]{5}$$

s.a.m