شهرزاد شوشتری

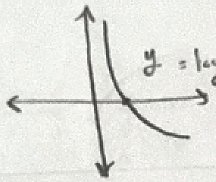
$\log_n m = a$, $\log_{mn} m^2 n = b \to 2\log_{mn} m + \log_{mn} n \to \dfrac{2}{\cancel{\log_m m}+\dfrac{1}{\log_n m}} + \dfrac{1}{\log_n n + \cancel{\log_n n}}$

$\dfrac{2}{\log_m mn} \to \dfrac{2}{\log_m m + \dfrac{1}{\log_n m}}$

$\to \dfrac{2}{\underbrace{1+\frac{1}{a}}_{\frac{a+1}{a}}} + \dfrac{1}{a+1} \to \dfrac{2a}{a+1} + \dfrac{1}{a+1} = \dfrac{2a+1}{a+1} \to 1 + \dfrac{a}{a+1} \to \left[1 + \dfrac{a}{a+1}\right] = 1$

(2)

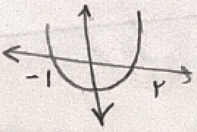
$y = \sqrt{\dfrac{x}{\log_{\frac{1}{2}} x}} \to \dfrac{x}{\log_{\frac{1}{2}} x} \geq 0$



$\to D_f = \varnothing$

$y = \dfrac{\log_4 (x^2 - x - 2)}{\sqrt{x^2-1}+1} \to x^2 - 1 \geq 0 \to x^2 \geq 1 \to x \geq 1 , x \leq -1$ ① $x^2 - x - 2 > 0 \to \Delta > 0$

$D_f = (-\infty, -1) \cup (2, +\infty)$



$\to x < -1 , x > 2 \to \cap \to (-\infty, -1) \cup (2, +\infty)$

(3)

$2\log_{3^2} a + \log_a 3 = 2 \to \log_3 a + \dfrac{1}{\log_3 a} = 2 \quad \log_3 a = t \quad t^2 + 1 = 2t \to t^2 - 2t + 1 = 0$

$(t-1)^2 = 0 \quad t = 1 \to \log_a 3 = 1 \quad \boxed{a = 3}$

(4)

$(\log 5 - \log 3)x^2 + (2\log 3)x - (\log 3 + \log 5) \qquad \log_{10} 10 = 1 = \underbrace{\log 2}_{0.3} + \log 5 \to \log 5 = 0.7$

$(0.7 - 0.4)x^2 + 0.8x - (0.4 + 0.7)$

$0.3x^2 + 0.8x - 1.1 \xrightarrow{\times 10} 3x^2 + 8x - 11 \to x^2 + 8x - 33 \to (x+11)(x-3) = 0$

$-\dfrac{11}{3} - 1 = -\dfrac{14}{3} \to \left|-\dfrac{14}{3}\right| = \dfrac{14}{3} \qquad x = -\dfrac{11}{3} \quad x = \dfrac{3}{3} = 1$

شهزاد شوشتری

۴)

$\log_{۱۵}^{۹} = \frac{\log_{۲}^{۹}}{\log_{۲}^{۱۵}} \to \frac{\log_{۲}^{۲} + \log_{۲}^{۳}}{\log_{۲}^{۳} + \log_{۲}^{۵}} = \frac{۱+۱/۶}{۱/۶+۲/۴} = \boxed{/۶۵}$

$\log_{۳}^{۵} \times \log_{۲}^{\cancel{۳}} = \log_{۲}^{۵} \to \log_{۳}^{۵} \times \log_{۲}^{۳} = ۱/۵ \times ۱/۶ = ۲/۴$

۵)

$\log_{۲}^{۷} = ۲/۸ \qquad \log_{۵}^{۲} = ۰/۵ \qquad \log_{۱۴}^{۱۰}$

$\log_{۵}^{۲} = \frac{\log_{۱۰}^{۲}}{\log_{۱۰}^{۱۰} - \log_{۱۰}^{۲}} = \frac{\log ۲}{۱-\log ۲} = \frac{۱}{۲} \to \log ۲ = \frac{۱}{۳}$

$\log_{۲}^{۷} = \frac{\log_{۱۰}^{۷}}{\frac{۱}{۳}} = \frac{۲۸}{۱۰} \to \log ۷ = \frac{۱۴}{۵} \to \log_{۱۴}^{۱۰} = \frac{۱}{\log_{۱۰}^{۱۴}} = \frac{۱}{\log ۲ \times ۷} = \frac{۱}{\frac{۱}{۳}+\frac{۱۴}{۱۵}} = \frac{۱۵}{۱۹}$

۷)

$\log_{۸}^{۱۸} = m \to \frac{۱}{۳}\cancel{\log_{۲}^{۲}} + \frac{۲}{۳}\log_{۲}^{۳} = m \to \frac{۲}{۳}\log_{۲}^{۳} = m - \frac{۱}{۳} \to \frac{۱}{۲}\log_{۲}^{۳} = \frac{\frac{۳m-۱}{\cancel{۳}}}{\frac{۴}{\cancel{۳}}}$

$\log_{۴}^{۱۲} = \frac{۱}{۲}\cancel{\log_{۲}^{۴}} + \frac{۱}{۲}\log_{۲}^{۳} = ۱ + \frac{۳m-۱}{۴} \to \frac{۳m+۳}{۴} = \frac{۳(m+۱)}{۴}$

۸)

$(۰/۴)^{۲x-۱} = \left(\frac{۱۲۵}{۸}\right)^{x^۲} \to \log_{۸}^{۹x+۱} = ?$

$\left(\frac{۲}{۵}\right)^{۲x-۱} = \left(\frac{۵}{۲}\right)^{۳x^۲} \to \left(\frac{۵}{۲}\right)^{۱-۲x} = \left(\frac{۵}{۲}\right)^{۳x^۲} \to ۳x^۲ = -۲x+۱ \to ۳x^۲+۲x-۱=۰$

$۳x^۲+۲x-۱=۰ \to x=-۱$ غ ق ق

$x = \frac{۱}{۳} \qquad \log_{۸}^{۴} = \frac{۲}{۳}\log_{۲}^{۲} = \frac{۲}{۳}$

$\log_{۲}^{۳} = a \qquad \log_{۸}^{b} = \frac{۲}{۳}(۱+a) \to \log(۳b-۸)$

$\log_{۸}^{b} = \frac{۲}{۳}+\frac{۲}{۳}\log_{۲}^{۳} \to \log_{۸}^{b} = \frac{۲}{۳}+\log_{۸}^{۹} \to \log_{۸}^{b} - \log_{۸}^{۹} = \frac{۲}{۳} \to \log_{۸}^{\frac{b}{۹}} = \frac{۲}{۳}$

$۸^{\frac{۲}{۳}} = \frac{b}{۹} \to \frac{b}{۹} = ۴ \to b = ۳۶ \qquad \log(۳\times۳۶-۸) = \log_{۱۰}^{۱۰۰} = ۲$

شهرزاد شریعتی

$-۴ax^{۲}+bx+\frac{۱}{۲}c=۰ \rightarrow \frac{۱}{\alpha}+\frac{۱}{\beta}=\log_{۱۰}^{۴}$ $\left(\frac{۱}{\sqrt[۴]{۲}}\right)^{\frac{c}{a}}$

$\frac{۱}{\frac{b}{۴a}}=\log^{۴} \rightarrow \frac{۴a}{b}=\log_{۱۰}^{۴}$ $\frac{a}{b}=\frac{\log^{۴}}{۴}=\frac{۲\log^{۲}}{۴}=\frac{\log^{۲}}{۲}$

$۲\left(\frac{\log^{۲}}{۲}\right)=۱+\frac{c}{b} \rightarrow \frac{c}{b}=-۱+\log^{۲}$ $\frac{c}{b}=-۱+\log^{۲}$

$\frac{a}{b}=\frac{\log^{۲}}{۲} \rightarrow \frac{c}{a}=\frac{-۱+\log^{۲}}{\frac{\log^{۲}}{۲}}=\frac{\log\frac{۲}{۱۰}}{\frac{۱}{۲}\log^{۲}}$

$=\frac{\log\frac{۱}{۵}}{\frac{۱}{۲}\log^{۲}}=\frac{\log\frac{۱}{۵}}{\log\sqrt{۲}}=\log_{\sqrt{۲}}\frac{۱}{۵} \rightarrow \left(\frac{۱}{\sqrt[۴]{۲}}\right)^{\frac{c}{a}} \rightarrow \left(\frac{۱}{\sqrt[۴]{۲}}\right)^{\log_{\sqrt{۲}}\frac{۱}{۵}} \rightarrow \frac{۱}{۵}^{\log_{\sqrt{۲}}\frac{۱}{\sqrt[۴]{۲}}}$

$\left(\frac{۱}{۵}\right)^{-\frac{۱}{۴}} \rightarrow ۵^{\frac{۱}{۴}} \Rightarrow \sqrt[۴]{۵}$