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لیانا ابراهیمی تکلیف شماره ۲۳

۱_ $n^a = m$

$\log_{mn}^{m^2n} = b \rightarrow \log_{n^a \times n}^{n^{2a} \times n} = b \Rightarrow \log_{n^{a+1}}^{n^{2a+1}} = b \rightarrow \frac{2a+1}{a+1} \log_n^n = b$

$\Rightarrow [\frac{a+a+1}{a+1}] = [b] \rightarrow [1 + \frac{a}{a+1}] = [b] \Rightarrow 1 + [\frac{a}{a+1}] = [b]$

$\Rightarrow 0 < \frac{a}{a+1} < 1 \rightarrow \boxed{[b] = 1}$

۲_

الف) $y = \sqrt{\frac{x}{\log_{\frac{1}{2}}^{x}}}$ $x > 0$ ① $\log_{\frac{1}{2}} x \neq 0 \rightarrow x \neq 1$ ②

$\boxed{D_f = (0, 1)}$

ب) $y = \frac{\log_{4}^{(x^2-x-2)}}{\sqrt{x^2-1} + 1}$ $x^2 - x - 2 > 0$

$x = -1$

$x = -\frac{c}{a} = 2$

$\sqrt{x^2-1} \neq -1$

$x^2 - 1 \geq 0 \rightarrow (x-1)(x+1) \geq 0 \rightarrow$ $\boxed{D_f = (-\infty, -1) \cup (2, +\infty)}$

۳_ $2\log_x^a + \log_a^{x^{\frac{1}{2}}} \rightarrow 2\log_x^a + \frac{1}{2}\log_a^x \Rightarrow 2\log_x^a + 2\log_x^a = 2$

$\log_x^a = \frac{2}{4} \rightarrow a = x^{\frac{1}{2}} \rightarrow x = 9 \rightarrow a = 9^{\frac{1}{2}} \rightarrow \sqrt{9} \rightarrow \boxed{a = 3}$

۴_ $(\log 5 - \log 3)\, x^2 + (2\log 3)\, x - (\log 3 + \log 5) = 0$ جمع ضرایب ۰ $x = 1$, $x = \frac{c}{a}$

$\frac{c}{a} = \frac{-\log 3 - \log 5}{\log 5 - \log 3} = \frac{-0{,}4 - \log 10 + \log 2}{\log 10 - \log 2 - 0{,}4} = \frac{-0{,}4 - 1 + 0{,}3}{1 - 0{,}3 - 0{,}4} = \frac{-1{,}1}{0{,}3} = -\frac{11}{3}$

$x_1 - x_2 = 1 - (-\frac{11}{3}) = \frac{14}{3}$

$\log_5^2 \times \log_2^7 = 0{,}5 \times 2{,}8 \Rightarrow \log_5^7 = 1{,}4$ ـ5

$\log_{14}^{10} = \dfrac{\log_5^2 + \log_5^5}{\log_5^2 + \log_5^7} = \dfrac{0{,}5+1}{0{,}5+1{,}4} = \dfrac{1{,}5}{19} = \dfrac{15}{19}$

ـ6

$\log_{15}^{6} = \dfrac{\log_2^6}{\log_2^{15}} \Rightarrow \dfrac{\log_2^2 + \log_2^3}{\log_2^3 + \log_2^5} = \dfrac{1+1{,}6}{1{,}6+2{,}4} = \dfrac{2{,}6}{4} = \dfrac{1{,}3}{2} = \dfrac{65}{100}$

$\log_4^{12} = \dfrac{\log_8^{12}}{\log_8^4} \Rightarrow \log_8^{12} = \log_8^4 + \log_8^3 \to \frac{2}{3} + \frac{1}{3}\log_2^3$ ـ7

$\log_8^4 = \frac{2}{3}\log_2^2 = \frac{2}{3}$

$\log_8^{18} = m \to \log_8^9 + \log_8^2 = m \Rightarrow \frac{2}{3}\log_2^3 + \frac{1}{3}\log_2^2 = m$

$\Rightarrow \frac{2}{3}\log_2^3 + \frac{1}{3} = m$

$\log_2^3 = \dfrac{3m-1}{2}$ $\qquad \log_4^{12} = \dfrac{\frac{2}{3}+\frac{1}{3}\log_2^3}{\frac{2}{3}} = \dfrac{\frac{2}{3}+\frac{1}{3}\left(\frac{3m-1}{2}\right)}{\frac{2}{3}} = \frac{3}{4}(m+1)$

ـ8

$\left(\frac{4}{10}\right)^{2x-1} = \left(\frac{5^3}{2^3}\right)^{x^2} \Rightarrow \left(\frac{2}{5}\right)^{2x-1} = \left(\frac{5}{2}\right)^{3x^2} \Rightarrow 2x-1 = -3x^2$

$3x^2 + 2x - 1 = 0 \to x^2 + 2x - 3 = 0 \to \left(x + \frac{3}{3}\right)\left(x - \frac{1}{3}\right) = 0$

$x = -1$ غ‌ق‌ق $\to 9x + 1 = -9 + 1 = -8$

$x = \frac{1}{3}$ ق‌ق $\to \log_{2^3}^{(3+1)} = \log_{2^3}^{2^2} = \frac{2}{3}\log_2^2 = \frac{2}{3}$

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$\frac{1}{3}\log\frac{b}{2} = \frac{2}{3}(a+1) \rightarrow \log\frac{b}{2} = 2a+2$

$2^{2a+2} = b \qquad (2^a)^2 \times 4 = 36 \rightarrow b = 36$

$\log(3b-8) \rightarrow \log(100) = 2$

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$S = \frac{-b}{-4a} = \frac{b}{4a} = \log 4 \rightarrow \frac{a}{b} = \frac{\log 4}{4} \rightarrow \frac{2\log 2}{4} = \frac{\log 2}{2}$

$2a = b+c \rightarrow \frac{2a}{b} = 1 + \frac{c}{b} \rightarrow a/b = \frac{\log 2}{2} \rightarrow 2\left(\frac{\log 2}{2}\right) = 1 + \frac{c}{b}$

$\frac{c}{b} = -1 + \log 2 \xrightarrow{\div 2} \frac{c}{a} = \frac{-1+\log 2}{\frac{\log 2}{2}} \rightarrow \frac{c}{a} = \frac{\log 2 - \log 10}{\frac{1}{2}\log 2} =$

$\frac{\log\frac{2}{10}}{\frac{1}{2}\log 2} = \frac{\log\frac{1}{5}}{\frac{1}{2}\log 2} \rightarrow \frac{\log\frac{1}{5}}{\log\sqrt{2}} = \log_{\sqrt{2}}\frac{1}{5} \rightarrow \left(\frac{1}{\sqrt[4]{2}}\right)^{4a}$

$= \left(\frac{1}{5}\right)^{\log_{\sqrt{2}}\frac{1}{\sqrt[4]{2}}} = \left(\frac{1}{5}\right)^{\log_{2^{\frac{1}{2}}} 2^{-\frac{1}{4}}} = \frac{1}{5}\left(\frac{-\frac{1}{4}}{\frac{1}{2}}\right)\log_2 2 = \left(\frac{1}{5}\right)^{-\frac{1}{4}}$

$= \boxed{\sqrt[4]{5}}$

Elipon