به نام خداوند دانش

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①

$\log_n m = a \qquad \log_{mn} m^2 n = b \qquad a>0 \qquad [b]=?$

$\log_m n = \frac{1}{a}$

$\log_{mn} m^2 n = \log_{mn} m^2 + \log_{mn} n = 2\log_{mn} m + \log_{mn} n = \frac{2a}{a+1} + \frac{1}{a+1}$

$\log_{mn} n = \frac{1}{\log_n mn} = \frac{1}{\log_n n + \log_n m} = \frac{1}{a+1}$

$= \frac{2a+1}{a+1} = 1 + \underbrace{\frac{a}{a+1}}_{\text{کوچکتر از واحد}} = b$

$\log_{mn} m = \frac{1}{\log_m mn} = \frac{1}{\log_m m + \log_m n} = \frac{1}{1+\frac{1}{a}} = \frac{1}{\frac{a+1}{a}} = \frac{a}{a+1}$

$\Rightarrow [b] = \left[1+\frac{a}{a+1}\right] = 1$

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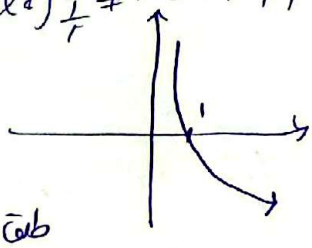
②

الف) $y = \sqrt{\frac{x}{\log_{\frac12} x}}$ به خاطر $\log$: $x>0$ $\log_{\frac12} x \neq 0 \Rightarrow x \neq 1$

$\frac{x}{\log_{\frac12} x} \geq 0$



$D_f = (0,1)$

x به خاطر دامنه‌ی log بزرگتر از صفر است. در نتیجه مخرج نیز باید مثبت باشه $\Longrightarrow$ طبق نمودار در بازه‌ی $(0,1)$ نمودار $\oplus$ می‌شود

ب) $y = \frac{\log_{\cancel{x}} (x^2-x-2)}{\sqrt{x^2-1}+1}$ به خاطر $\log$: $x^2-x-2>0 = (x-2)(x+1)>0$

① $x^2-1 \geq 0 = (x+1)(x-1) \geq 0$

	-1		1	
$+$		$-$		$+$

②

	-1		2	
$+$		$-$		$+$

$\cap$

$D_f = (-\infty,-1) \cup (2,+\infty)$

③ $\sqrt{x^2-1}+1 \neq 0 \Rightarrow \sqrt{x^2-1} \neq -1$

همواره برقرار است

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③

$2\log_x a + \log_a \sqrt{x} = 2 \qquad x=9 \qquad a=?$

$\log_x a = t \qquad \frac{1}{\log_x a} = \log_a x = \frac{1}{t}$

$\log_a \sqrt{x} = \frac{1}{2}\log_a x = \frac{1}{2t}$

$2t + \frac{1}{2t} = 2 \Rightarrow \frac{4t^2+1}{2t} = 2 \Rightarrow 4t^2-4t+1=0$ $(\div 4)$

$t^2-4t+4 = (t-2)^2 = 0$

$t = \frac{2}{4} = \frac{1}{2}$

$\log_x a = \frac{1}{2} \Rightarrow \log_9 a = \frac{1}{2} \Rightarrow \sqrt{9} = \boxed{3 = a}$

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$\log 2 = 0.3 \quad \log 3 = 0.4$

$(\log\frac{5}{3})x^2 + (\log 9)x - \log 15 = 0$

$$\frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{(\log 9)^2 - 4\times(\log\frac{5}{3})(-\log 15)}}{|\log\frac{5}{3}|} = \frac{\sqrt{(0.8)^2 + 4\times 0.3\times 1.1}}{0.3} = \frac{\sqrt{0.64+1.32}}{0.3}$$

$$= \frac{\sqrt{1.96}}{0.3} = \frac{1.4}{0.3} = \boxed{\frac{14}{3}}$$

$\log_{10} 9 = 2\log_{10} 3 = 2\times 0.4 = \boxed{0.8}$

$\log\frac{5}{3} = \log 5 - \log 3 = 0.7 - 0.4 = 0.3$

$\log 5 = \log\frac{10}{2} = \log 10 - \log 2 = 1 - 0.3 = 0.7$

$\log 15 = \log 5\times 3 = \log 5 + \log 3 = 0.7 + 0.4 = 1.1$

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$\log_2 7 = 2.8 \quad \log_5 2 = 0.5 \qquad \log_{14} 10 = \frac{1}{\log_{10} 14} = \frac{1}{\log_{10} 2 + \log_{10} 7} = \frac{1}{\frac{5}{15}+\frac{14}{15}}$

$$= \frac{1}{\frac{19}{15}} = \boxed{\frac{15}{19}}$$

$\log_2 5 = \frac{1}{\log_5 2} = \frac{1}{\frac{1}{2}} = 2$

$\log_2 10 = \underbrace{\log_2 5}_{2} + \underbrace{\log_2 2}_{1} = 3 \Rightarrow \log_{10} 2 = \frac{1}{3}$

$\log_{10} 7 = \frac{\log_2 7}{\log_2 10} = \frac{2.8}{3} = \frac{\frac{28}{10}}{3} = \frac{28}{30} = \frac{28}{30} = \frac{14}{15}$

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$\log_3 5 = 1.5 \quad \log_2 3 = 1.6 \qquad \log_{15} 6 = \log_{15} 2 + \log_{15} 3 = \frac{5}{20} + \frac{8}{20} = \boxed{\frac{13}{20}}$

$\log_3 2 = \frac{1}{1.6} = \frac{10}{16} = \frac{5}{8}$

$\log_{15} 2 = \frac{1}{\log_2 15} = \frac{1}{\log_2 3 + \log_2 5} = \frac{1}{1.6 + 2.4} = \frac{1}{4}$

$\log_{15} 3 = \frac{1}{\log_3 15} = \frac{1}{\underbrace{\log_3 5}_{1.5} + \cancel{\log_3 3}} = \frac{1}{2.5} = \frac{10}{25} = \frac{2}{5}$

$\log_2 5 = \frac{\log_3 5}{\log_3 2} = \frac{1.5}{\frac{5}{8}} = \frac{\frac{15}{10}}{\frac{5}{8}} = \frac{12}{5} = 2.4$

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$\log_{8}^{18} = m \qquad \log_{4}^{12} = \log_{4}^{3} + \cancel{\log_{4}^{4}}_{1} = \frac{3m-1}{4} + \frac{4}{4} = \frac{3m+3}{4}$

$m = \log_{8}^{18} = \log_{8}^{3^2\times 2} = 2\log_{8}^{3} + \frac{1}{3}\cancel{\log_{2}^{2}}$

$2\log_{8}^{3} = m - \frac{1}{3} = \frac{3m-1}{3} \Rightarrow \log_{8}^{3} = \frac{3m-1}{6}$

$\log_{8}^{3} = \log_{2^3}^{3} = \frac{1}{3}\log_{2}^{3} = \frac{3m-1}{6} \Rightarrow \log_{2}^{3} = \frac{3m-1}{2}$

$\frac{1}{2}\log_{2}^{3} = \log_{4}^{3} = \frac{3m-1}{4}$

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$(0/4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \qquad \log_{8}^{(9x+1)} = \cancel{\log_{8}^{-9+1}}$ تعریف نشده $\quad 1 \quad \log_{8}^{\frac{1}{3}\times 9+1}$

$= \log_{8}^{4} = \frac{2}{3}\log_{2}^{2} = \boxed{\frac{2}{3}}$

$\left(\frac{2^2}{10}\right)^{2x-1} = \frac{2^{4x-2}}{10^{2x-1}} = \frac{5^{3x^2}}{2^{3x^2}}$

$\left(\frac{5^3}{2^3}\right)^{x^2} \Rightarrow 2^{3x^2+4x-2} = 2^{2x-1} \times 5^{2x-1} \times 5^{3x^2}$

$\Rightarrow 2^{3x^2+2x-1} = 5^{3x^2+2x-1} \Rightarrow 3x^2+2x-1 = 0$

$x^2 + 2x - 3 = 0, \ (x+3)(x-1) = 0, \ \div 3$

$\frac{-3}{3} = -1 \quad \frac{1}{3}$

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$\log_{2}^{3} = a \qquad \log_{8}^{b} = \frac{2}{3}(1+a) \qquad \frac{1}{3}\underbrace{\log_{2}^{3}}_{a} = \log_{8}^{3} = \frac{1}{3}a$

$\log(3b-8)$

$\log_{8}^{b} = \frac{2}{3}\left(\log_{2}^{2} + \log_{2}^{3}\right) = \frac{2}{3}\log_{2}^{2} + \frac{2}{3}\log_{2}^{3} = \log_{8}^{4} + \log_{8}^{9} = \log_{8}^{36}$

$\boxed{b = 36}$

$\log(3b-8) = \log(3\times 36 - 8) = \log^{108-8} = \log^{100} = \boxed{2}$

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$-4ax^2 + bx + \frac{1}{2}c = 0 \qquad \frac{a}{b} = \frac{1}{2}\log^{2} \Rightarrow \frac{b}{a} = \frac{2}{\log^{2}}$

$\frac{-b}{-4a} = \frac{b}{4a} \Rightarrow \frac{4a}{b} = \log^{4} \qquad \left(\frac{1}{2^{\frac{1}{8}}}\right)^{\frac{c}{a}} = \frac{1}{2^{\frac{1}{8}\times\frac{c}{a}}} = 2^{-\frac{1}{8}(2-\log_{2}^{10})}$

$= \frac{2^{-\frac{1}{4}}}{2^{-\frac{1}{8}\log_{2}^{10}}}$

$b + c = 2a \Rightarrow b = 2a - c \qquad \frac{4a}{2a-c} = \log^{4}$

$\frac{c}{a} = \frac{2a-b}{a} = 2 - \frac{b}{a} = 2 - \frac{2}{\log^{2}} = (2 - \log_{2}^{100})$

$\frac{2^{-\frac{1}{4}}}{100^{-\frac{1}{8}}} = (0/2)^{-\frac{1}{4}} = 5^{\frac{1}{4}} = \sqrt[4]{5}$