نام و نام خانوادگی: آلا ادریسی   تکلیف شماره ۲۳   کلاس یازدهم دختر

۱ $\log_{mn}^{m^2 n} = b \to \frac{\log_n^{m^2 n}}{\log_n^{mn}} = \frac{\log_n^{m^2}+\log_n^n}{\log_n^m+\log_n^n} = \frac{2\times\log_n^m+1}{\log_n^m+1} \xrightarrow{\log_n^m = a} b = \frac{2a+1}{a+1}$

$[b] = \left[\frac{2a+1}{a+1}\right] = \left[\frac{a+a+1}{a+1}\right] = \left[1+\frac{a}{a+1}\right] = 1+\underbrace{\left[\frac{a}{a+1}\right]}_{0} = 1$

الف) $\frac{x}{\log_{1/2}^x} > 0$: ریشه‌ها : $x=0$ ، $\log_{1/2}^x = 0 \Rightarrow x=1$ $\Big\}\Longrightarrow$ $\begin{array}{c|c|c} & 0 & 1 \\ - & + & - \end{array}$ $\Rightarrow x\in[0,1)$ (1)

همچنین دامنه log : $x>0$ (2) $\xrightarrow{(1)\cap(2)}$ $(0,1)$ دامنه

۲ ب) ① $x^2-x-2>0 \to (x-2)(x+1)>0 \Rightarrow$ $\begin{array}{c|c|c} & -1 & 2 \\ + & - & + \end{array}$ $\Rightarrow x\in(-\infty,-1)\cup(2,+\infty)$

② $x^2-1>0 \to (x-1)(x+1)>0 \Rightarrow$ $\begin{array}{c|c|c} & -1 & 1 \\ + & - & + \end{array}$ $\Rightarrow x\in(-\infty,-1)\cup(1,+\infty)$

③ $\sqrt{x^2-1}\neq -1$ → همواره برقرار است.

$\xrightarrow{①\cap②}$ $(-\infty,-1)\cup(2,+\infty)$ دامنه

$x=9 \Rightarrow 2\log_9^a + \frac{1}{2}\log_a^9 = 2 \to 2\underbrace{\log_9^a}_{t} + \frac{1}{2\log_9^a} = 2 \xrightarrow{\times 2t} 4t^2+1=4t$

۳ $\Rightarrow 4t^2-4t+1=0 \Rightarrow (2t-1)^2=0 \to t=\frac{1}{2} \Rightarrow \log_9^a = \frac{1}{2} \to a = 9^{1/2} = \sqrt{9} = 3$

$* \log 5 = \log\frac{10}{2} = \log 10 - \log 2 = 1-\log 2$

$\Rightarrow (\log\frac{5}{6})x^2 + (\log 9)x - \log 15 = 0 \to (1-\log 2-\log 3)x^2 + (2\log 3)x - (\log 3 + 1 - \log 2) = 0$

۴ $\to (\frac{10}{10} - \frac{3}{10} - \frac{4}{10})x^2 + (2\times\frac{4}{10})x - (\frac{4}{10}+\frac{10}{10}-\frac{3}{10}) = 0 \to \frac{3}{10}x^2+\frac{8}{10}x-\frac{11}{10}=0$

$\xrightarrow{\times 10} 3x^2+8x-11=0 \xrightarrow{a+b+c=0} x_1=1 \quad x_2=-\frac{11}{3} \xrightarrow{\text{اختلاف}} \frac{3}{3}+\frac{11}{3}=\frac{14}{3}$

$\log_{14}^{10} = \frac{\log_2^{10}}{\log_2^{14}} = \frac{\log_2^5+\log_2^2}{\log_2^7+\log_2^2} = \frac{2+1}{2.8+1} = \frac{3}{3.8} = \frac{30}{38} = \frac{15}{19}$

۵

$* \log_5^2 = \frac{5}{10} = \frac{1}{\log_2^5} \Rightarrow \log_2^5 = \frac{10}{5} = 2$

$\log_{15}^6 = \frac{\log_3^6}{\log_3^{15}} = \frac{\log_3^2+\log_3^3}{\log_3^5+\log_3^3} = \frac{1+\frac{10}{16}}{1+\frac{15}{10}} = \frac{\frac{26}{16}}{\frac{25}{10}} = 0.65$

۶

$* \log_2^3 = \frac{16}{10} \Rightarrow \frac{1}{\log_3^2} = \frac{16}{10} \Rightarrow \log_3^2 = \frac{10}{16}$

7 $\log_8 18 = \log_{2^3} 3^2\times 2 = \log_2 3^{\frac{2}{3}} + \log_2 2^{\frac{1}{3}} = \frac{2}{3}\log_2 3 + \frac{1}{3} = m \Rightarrow (m-\frac{1}{3})\frac{3}{2} = \log_2 3$

$\rightarrow \log_4 12 = \log_{2^2} 3\times 2^2 = \frac{1}{2}\log_2 3 + 2\log_2 2 = \frac{3}{4}(m-\frac{1}{3}) + 2 = \frac{3}{4}m - \frac{1}{4} + \frac{8}{4} = \frac{3}{4}m + \frac{7}{4}$]

$(\frac{4}{10})^{2x-1} = (\frac{125}{8})^{x^2}$ $\rightarrow (\frac{2}{5})^{2x-1} \rightarrow (\frac{5}{2})^{1-2x}$; $\rightarrow ((\frac{5}{2})^3)^{x^2} \rightarrow (\frac{5}{2})^{3x^2}$ $\longrightarrow 1-2x = 3x^2$

8 $\rightarrow 3x^2+2x-1=0 \xrightarrow{a+c=b} x_1=-1 \;,\; x_2=+\frac{1}{3}$

غ ق ق ⇐ در دامنه $\log_8(9x+1)$ نیست

$\Rightarrow \log_8 9x+1 = \log_8 4 = \log_{2^3} 2^2 = \frac{2}{3}$]

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9 $\log_8 b = \frac{2}{3}(\cancel{1} + \log_2 3) \Rightarrow \log_8 b = \log_8 27 \Rightarrow b=27 \Rightarrow \log(4b-8) = \log 100 = 2$]

$\log_2 2 \longleftrightarrow \log_2 4$

$\frac{1}{S} = \log 4 \Rightarrow S = \frac{1}{\log 4} = \frac{b}{4a} \Rightarrow b = \frac{4a}{\log 4}$

$a = \frac{b+c}{2} \Rightarrow c = 2a - b = 2a - \frac{\cancel{4}^{2}a}{\cancel{2}\log 2} = 2a(1-\frac{1}{\log 2}) = 2a(\log_2 2 - \log_2 10)$

10 $c = 2a(-\log_2 5) \Rightarrow \frac{c}{a} = -2\log_2 5 = -\log_2 25$

$\rightarrow (\frac{1}{\sqrt[8]{2}})^{c/a} = 2^{-\frac{1}{8}\times\frac{c}{a}} = 2^{-\frac{1}{8}\times(-\log_2 25)} = 2^{\log_{2^8} 25} = 25^{\frac{1}{8}} = \sqrt[8]{25} = \sqrt[4]{5}$]