نام و نام خانوادگی: آیلا دارابی   تکلیف شماره ۲۳   کلاس یازدهم دختر C

۱۹٫۵

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$\log_{mn} m^2 n = b \to \dfrac{\log_n m^2 n}{\log_n mn} = \dfrac{\log_n m^2 + \log_n n}{\log_n m + \log_n n} = \dfrac{2\times\log_n m + 1}{\log_n m + 1} \xrightarrow{\log_n m = a} b = \dfrac{2a+1}{a+1}$

$[b] = \left[\dfrac{2a+1}{a+1}\right] = \left[\dfrac{a+a+1}{a+1}\right] = \left[1 + \dfrac{a}{a+1}\right] = 1 + \left[\dfrac{a}{a+1}\right] = 1$

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الف) $\dfrac{x}{\log_{1/2} x} > 0$: ریشه‌ها $x = 0$ ، $\log_{1/2} x = 0 \Rightarrow x = 1$ $\Big\} \Longrightarrow$ (جدول تعیین علامت: 0 ، 1 ؛ $- \;|\; + \;|\; -$) $\Rightarrow x \in [0,1)$ (1)

$\log$ تعیین دامنه: $x > 0$ (2) $\xrightarrow{(1)\cap(2)}$ $(0,1)$ دامنه

ب) ① $x^2 - x - 2 > 0 \to (x-2)(x+1) > 0 \Rightarrow$ (جدول: -1 ، 2 ؛ $+ \;|\; - \;|\; +$) $\Rightarrow x \in (-\infty,-1)\cup(2,+\infty)$

② $x^2 - 1 > 0 \to (x-1)(x+1) > 0 \Rightarrow$ (جدول: -1 ، 1 ؛ $+ \;|\; - \;|\; +$) $\Rightarrow x \in (-\infty,-1)\cup(1,+\infty)$

③ $\sqrt{x^2-1} \neq -1 \to$ همواره برقرار است

$\xrightarrow{①\cap②}$ $(-\infty,-1)\cup(2,+\infty)$ دامنه

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$x = 9 \Rightarrow 2\log_9 a + \frac{1}{2}\log_a 9 = 2 \to 2\underbrace{\log_9 a}_{t} + \dfrac{1}{2\log_9 a} = 2 \xrightarrow{\times 2t} 4t^2 + 1 = 4t$

$\Rightarrow 4t^2 - 4t + 1 = 0 \Rightarrow (2t-1)^2 = 0 \to t = \frac{1}{2} \Rightarrow \log_9 a = \frac{1}{2} \to a = 9^{\frac{1}{2}} = \sqrt{9} = 3$

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* $\log 5 = \log\frac{10}{2} = \log 10 - \log 2 = 1 - \log 2$

$\Rightarrow (\log\frac{5}{3})x^2 + (\log 9)x - \log 15 = 0 \to (1 - \log 2 - \log 3)x^2 + (2\log 3)x - (\log 3 + 1 - \log 2) = 0$

$\to (\frac{10}{10} - \frac{3}{10} - \frac{4}{10})x^2 + (2\times\frac{4}{10})x - (\frac{4}{10} + \frac{10}{10} - \frac{3}{10}) = 0 \to \frac{3}{10}x^2 + \frac{8}{10}x - \frac{11}{10} = 0$

$\xrightarrow{\times 10} 3x^2 + 8x - 11 = 0 \xrightarrow{a+b+c=0} x_1 = 1 \quad x_2 = -\frac{11}{3} \xrightarrow{\text{اختلاف}} \frac{3}{3} + \frac{11}{3} = \frac{14}{3}$

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$\log_{14} 10 = \dfrac{\log_2 10}{\log_2 14} = \dfrac{\log_2 5 + \log_2 2}{\log_2 7 + \log_2 2} = \dfrac{2+1}{2.8+1} = \dfrac{3}{3.8} = \dfrac{30}{38} = \dfrac{15}{19}$

* $\log_5 2 = \frac{5}{10} = \dfrac{1}{\log_2 5} \Rightarrow \log_2 5 = \frac{10}{5} = 2$

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$\log_{15} 6 = \dfrac{\log_3 6}{\log_3 15} = \dfrac{\log_3 2 + \log_3 3}{\log_3 5 + \log_3 3} = \dfrac{1 + \frac{10}{14}}{1 + \frac{15}{10}} = \dfrac{\frac{24}{14}}{\frac{25}{10}} = 0.45$

* $\log_2 3 = \frac{14}{10} \Rightarrow \dfrac{1}{\log_3 2} = \frac{14}{10} \Rightarrow \log_3 2 = \frac{10}{14}$

7 $\log_8^{18} = \log_{2^3}^{3^2\times 2} = \log_{2^3}^{3^2} + \log_{2^3}^{2} = \frac{2}{3}\log_2^{3} + \frac{1}{3} = m \Rightarrow (m-\frac{1}{3})\frac{3}{2} = \log_2^{3}$ (1,5)

$\rightarrow \log_4^{12} = \log_{2^2}^{3\times 2^2} = \frac{1}{2}\log_2^{3} + 2\log_2^{2} = \frac{3}{4}(m-\frac{1}{3}) + 2 = \frac{3}{4}m - \frac{1}{4} + \frac{8}{4} = \frac{3}{4}m + \frac{7}{4}$

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8 $(\frac{4}{10})^{2x-1} = (\frac{125}{8})^{x^2}$; $(\frac{2}{5})^{2x-1} \rightarrow (\frac{5}{2})^{1-2x}$; $((\frac{5}{2})^3)^{x^2} \rightarrow (\frac{5}{2})^{3x^2}$ $\longrightarrow 1-2x = 3x^2$

$\rightarrow 3x^2+2x-1=0 \xrightarrow{a+c=b} x_1=-1 \ , \ x_2=+\frac{1}{3}$

(x_1: غ ق ق ← نیست $\log_8^{(9x+1)}$ در دامنه ; x_2: ✓)

$\Rightarrow \log_8^{9x+1} = \log_8^{4} = \log_{2^3}^{2^2} = \frac{2}{3}$

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9 $\log_8^{b} = \frac{3}{2}(\cancel{1} + \log_2^{3})$ ($\log_2^{2} \longleftrightarrow \log_2^{4}$) $\Rightarrow \log_8^{b} = \log_8^{28} \Rightarrow b = 27 \Rightarrow \log^{(4b-8)} = \log^{100} = 2$

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10 $\frac{1}{5} = \log^{4} \Rightarrow 5 = \frac{1}{\log 4} = \frac{b}{4a} \Rightarrow b = \frac{4a}{\log 4}$

$a = \frac{b+c}{2} \Rightarrow c = 2a - b = 2a - \frac{4a}{2\log 2} = 2a(1-\frac{1}{\log 2}) = 2a(\log_2^{2} - \log_2^{10})$

$c = 2a(-\log_2^{5}) \Rightarrow \frac{c}{a} = -2\log_2^{5} = -\log_2^{25}$

$\rightarrow (\frac{1}{\sqrt[8]{2}})^{c/a} = 2^{-\frac{1}{8}\times\frac{c}{a}} = 2^{-\frac{1}{8}\times(-\log_2^{25})} = 2^{\log_{2^8}^{25}} = 25^{\frac{1}{8}} = \sqrt[8]{25} = \sqrt[4]{5}$

$\frac{\log 18}{\log 8} \Rightarrow \frac{\log 3}{\log 2} = \frac{3m-1}{2}$ $\quad \log_4^{12} = 1 + \log_4^{3}$ ($\frac{\log 3}{2\log 2}$) $= 1 + \frac{1}{2}(\frac{3m-1}{2}) = \frac{3m+3}{4}$