نیایش شاه نظری — یازدهم تجربی — ۱۹٫۵ — تکلیف کد ۲۰

① $\frac{1}{x-1} = |x-1|$

$x=1 \rightarrow$ در معادله صدق نمی کند

$x>1 \rightarrow x^2-2x+1=1$

$x^2-2x=0 \rightarrow x=0$ غ ق ق ، $\boxed{x=2}$

۲

$x<1 \rightarrow -x^2+2x-1=1$

$-x^2+2x-2=0$

$x^2-2x+2=0 \rightarrow \Delta<0$ ریشه ندارد

معادله مورد نظر یک ریشه دارد.

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② الف) $\left|\frac{2x-1}{x}\right| < 2$

$-2 < \frac{2x-1}{x} < 2 \rightarrow \frac{2x-1}{x} < 2$

$\frac{-1}{x} < 0 \rightarrow \underline{x>0}$

$\downarrow$

$0 < \frac{4x-1}{x}$

x		0		$\frac{1}{4}$	
	$+$		$-$		$+$

$x \in (-\infty, 0) \cup (\frac{1}{4}, +\infty)$

$(0,+\infty)\cap\left((-\infty,0)\cup(\frac{1}{4},+\infty)\right)=\boxed{(\frac{1}{4},+\infty)}$ ۱,۵

ب) $\left|\frac{x-2}{2x-1}\right|>1 \to \frac{x-2}{2x-1}>1 \to \frac{-x-1}{2x-1}>0$

-1 (-3)		$\frac{1}{2}$	
$-$	$+$		$-$

$x\in(-1,\frac{1}{2})$

$\frac{x-2}{2x-1}<-1 \to \frac{3x-3}{2x-1}<0$

$\frac{1}{2}$		$\cancel{1}$ $\frac{1}{3}$	
$+$	$-$		$+$

$x\in(\frac{1}{2},1)$

$(-1,\frac{1}{2})\cup(\frac{1}{2},1)=\boxed{\cancel{(-1,1)}-\{\cancel{\frac{1}{2}}\}}$ $(-3,\frac{1}{3})$

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۳) $x^2<|x+6| \to x>-6 \to x^2-x-6<0$

-2		3	
$+$	$-$		$+$

$x\in(-2,3)$

$x<-6 \to x^2+x+6<0$

-3		2	
$+$	$-$		$+$

$(-\infty,-6)\cap(-3,2)=\varnothing$ ۲

$-2<x<3$

$\frac{3+(-2)}{2}=\frac{1}{2} \to -\frac{5}{2}<x-\frac{1}{2}<\frac{5}{2} \to \boxed{|x-\frac{1}{2}|<\frac{5}{2}}$ $\boxed{\alpha=\frac{1}{2}}$

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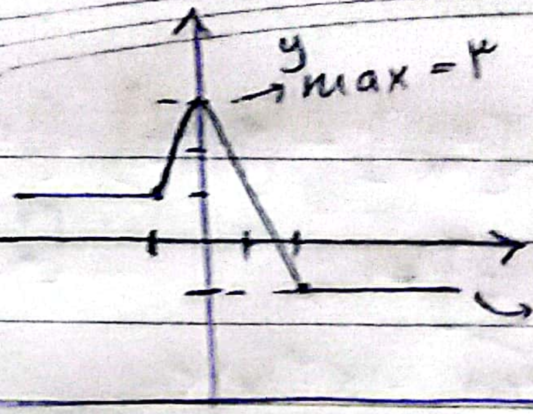
۴) $y=|x+1|-|2x|+|x-2|$ ۲

$x<-1$	$-1\le x<0$	$0\le x<2$	$x\ge 2$
$-x-1+2x+2-x$	$x+1+2x+2-x$	$x+1-2x+2-x$	$x+1-2x+x-2=-1$
1	$2x+3$	$-2x+3$	-1
1		3	-1



$\boxed{3-1=2}$

$y=\left|\frac{1}{2}x\right|-2 \longrightarrow y=\left|\frac{1}{2}(x+4)\right|-2 \longrightarrow y=\left|\frac{1}{2}x+2\right|-1$ (5)

$\left|\frac{1}{2}x\right|-2=\left|\frac{1}{2}x+2\right|-1 \to x>0 \to \frac{1}{2}x=\frac{1}{2}x+3 \Rightarrow$ غ ق ق 2

$-4<x<0 \to -\frac{1}{2}x-2=\frac{1}{2}x+1$

$\boxed{-3=x}$

$x<-4 \to -\frac{1}{2}x-2=-\frac{1}{2}x-3 \to$ غ ق ق

$x=0 \to -2=2-1$ ✗

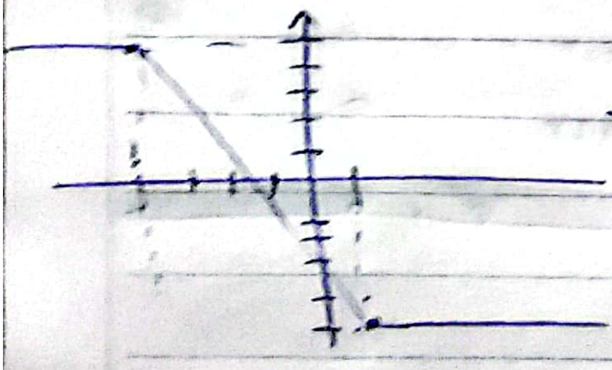
$x=-4 \to 0=-1$ ✗

9) $f(x)=\left[\,\left||1-x|-|x+4|\right|\,\right]=\left[\,\left||x-1|-|x+4|\right|\,\right]$ (4)

$|x-1|-|x+4| \to$

$x<-4$	$-4<x<1$	$x>1$
$1-x+x+4$ 5	$1-x-x-4$ $5 \quad -2x-3 \quad -5$	$x-1-x-4=$ -5

2



$\to \left[\,\left||x-1|-|x+4|\right|\,\right] \Rightarrow$

$R_f=\{-5,-4,-3,-2,-1,0,1,2,3,4,5\}$

ب) $f(x) = \frac{1}{x} - \left[\frac{x+1}{x}\right]$

$$\left[\frac{x+1}{x}\right] = \left[\frac{x}{x} + \frac{1}{x}\right] = \left[1 + \frac{1}{x}\right] = \left[\frac{1}{x}\right] + 1$$

$$\frac{1}{x} - \left(\left[\frac{1}{x}\right] + 1\right) = \underbrace{\frac{1}{x} - \left[\frac{1}{x}\right]} - 1 \Rightarrow$$

$0 \leq y < 1$ $\qquad -1 \leq f(x) < 0$

$$\boxed{R_f = [-1, 0)}$$

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(۷)

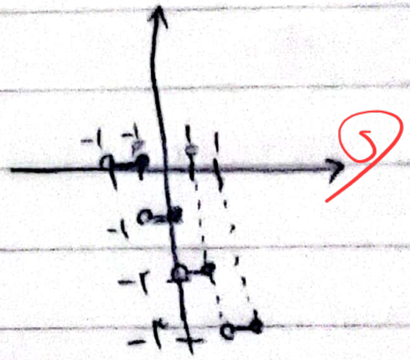
$y = [-2x] - 1 \qquad x \in (-1, 1)$

$x \in (-1, -\frac{1}{2}] \rightarrow -2x \in [1, 2) \rightarrow [-2x] - 1 = 0$

$x \in (-\frac{1}{2}, 0] \rightarrow -2x \in [0, 1) \rightarrow [-2x] - 1 = -1$

$x \in (0, \frac{1}{2}] \rightarrow -2x \in [-1, 0) \rightarrow [-2x] - 1 = -2$

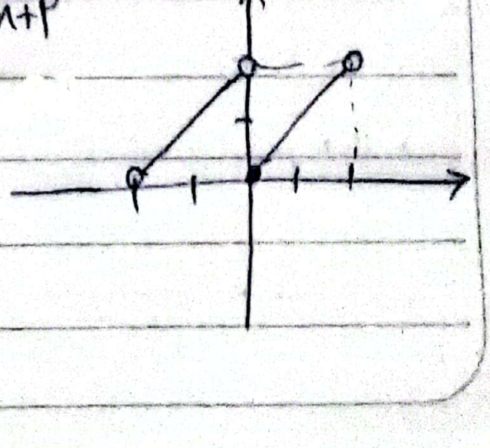
$x \in (\frac{1}{2}, 1) \rightarrow -2x \in (-2, -1) \rightarrow [-2x] - 1 = -3$



ب) $y = x - 2\left[\frac{x}{2}\right] \qquad x \in (-2, 2)$

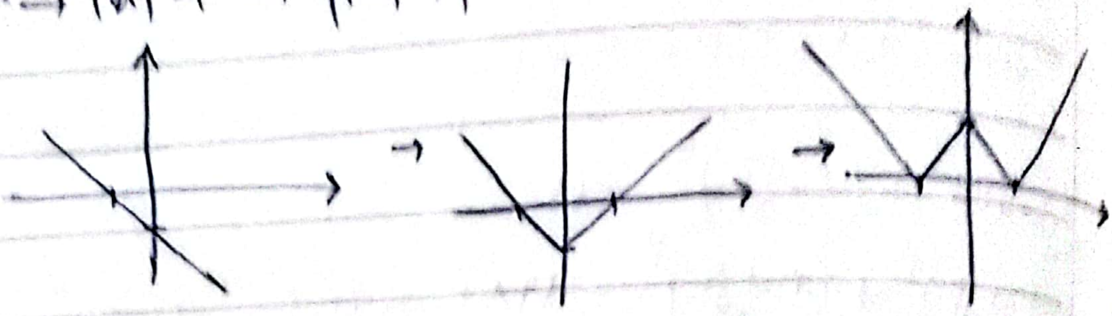
$x \in (-2, 0) \rightarrow y = x - 2(-1) \rightarrow y = x + 2$

$x \in [0, 2) \rightarrow y = x - 2(0) \Rightarrow y = x$

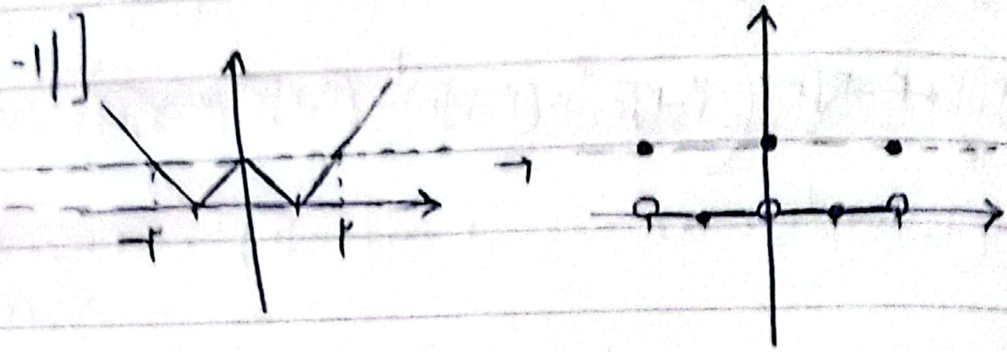


الف) $y = [\,||x|-1|\,]$ $\quad x \in [-۲, ۲]$ (۹

$x-1 \to |x|-1 \to ||x|-1|$

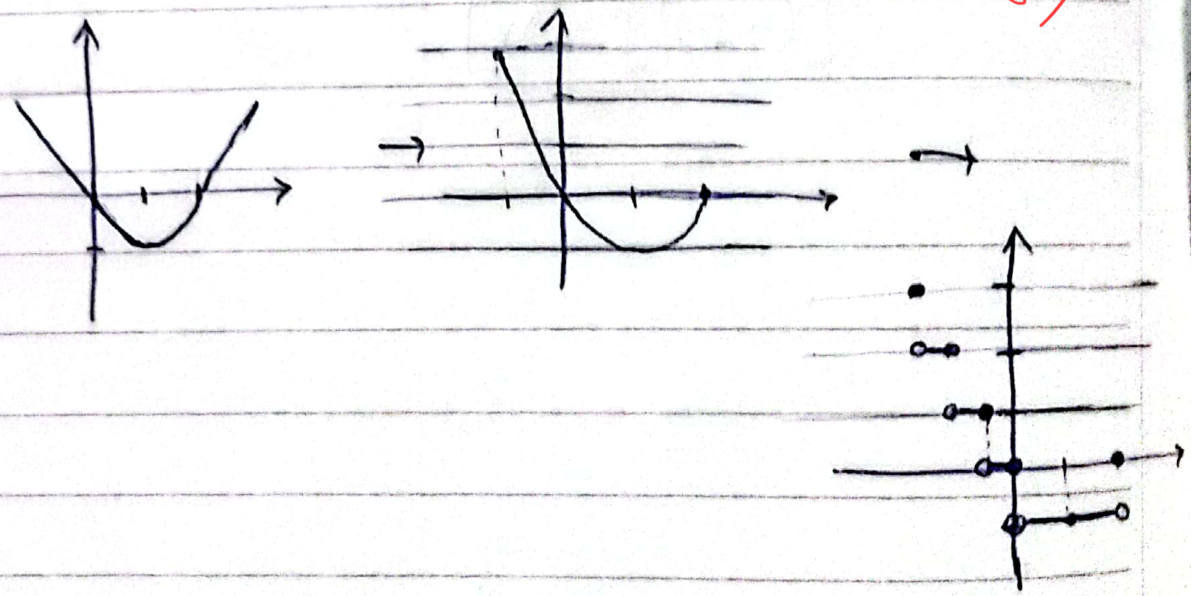


$[\,||x|-1|\,]$



ب) $y = [x^۲ - ۲x]$ $\quad x \in [-1, ۲]$

۵



$a + [b] = ۴,۲ \qquad [a] + Q_1 = a \qquad [a] + [b] + Q_1 = ۴,۲$ ①

$b - [a] = ۲,۴ \qquad [b] + Q_۲ = b \qquad \underline{Q_1 = ۰,۲}$

$\hookrightarrow [b] - [a] + Q_۲ = ۲,۴ \to \underline{Q_۲ = ۰,۴}$

۳

$[a] + [b] = ۴$

$[b] - [a] = ۲$

$$\begin{cases} [a]+[b]=۴ \\ [b]-[a]=۲ \end{cases} \Rightarrow ۲[b]=۶ \rightarrow [b]=۳ \rightarrow b=[b]+Q_۲=\boxed{۳,۴}$$

$$[a]=۱ \rightarrow a=[a]+Q_۱=\boxed{۱,۲}$$

$$a+b=۱,۲+۳,۴=\boxed{۴,۶}$$

(۱۰)

$$(۱+\sqrt{۲})^۶+(۱-\sqrt{۲})^۶=۱۹۸$$

$$(۳+۲\sqrt{۲})^۳-(۳-۲\sqrt{۲})^۳=۱۴۰\sqrt{۲} \rightarrow (۱+\sqrt{۲})^۶-(۱-\sqrt{۲})^۶=۱۴۰\sqrt{۲}$$

$$(۳+۲\sqrt{۲}-۳+۲\sqrt{۲})(۱۷+۱۲\sqrt{۲}+۱۷-۱۲\sqrt{۲}+۱)=۱۴۰\sqrt{۲}$$

$$۲(۱+\sqrt{۲})^۶=۱۹۸+۱۴۰\sqrt{۲} \rightarrow (۱+\sqrt{۲})^۶=۹۹+۷۰\sqrt{۲}$$

$$۷۰(۱,۴۱۴)=۹۸,۹۹-$$

$$\underbrace{۹۸,۹۹-}_{Q_۱}$$

$$(۱+\sqrt{۲})^۶=۹۹+۹۸+Q_۱$$

$$[(۱+\sqrt{۲})^۶]=\boxed{۱۹۷}$$