

$$\frac{1}{n-1} = |n-1| \begin{cases} n \geq 1 \rightarrow \frac{1}{n-1} = n-1 \rightarrow (n-1)^2 = 1 = n^2 + 1 - 2n = 1 \Rightarrow n(n-2) = 0 \\ n < 1 \rightarrow \frac{1}{n-1} = -(n-1) = -(n-1)^2 = 1 \Rightarrow -(n^2 + 1 - 2n) = 1 \Rightarrow n^2 - 2n + 1 = 0 \end{cases}$$

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الف) $\left| \frac{n-1}{n} \right| < 1 \rightarrow \frac{n-1}{n} - 1 < 0 \rightarrow \frac{n-1-n}{n} < 0 \rightarrow \frac{-1}{n} < 0 \rightarrow n > 0 \quad (1)$

$-1 < \frac{n-1}{n} < 1 \rightarrow -1 < \frac{n-1}{n} + 1 < 2 \rightarrow \frac{-n+1+n}{n} < 2 \rightarrow \frac{1}{n} < 2 \rightarrow n > \frac{1}{2} \quad (2)$

$(1) \cap (2) \rightarrow \left(\frac{1}{2}, +\infty \right)$

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ب) $\left| \frac{n-2}{n+1} \right| > 1$

$\frac{n-2}{n+1} - 1 > 0 \rightarrow \frac{n-2-n-1}{n+1} > 0 \rightarrow \frac{-2n-3}{n+1} > 0 \rightarrow (-2, -\frac{3}{2}) \quad (1)$

$\frac{n-2}{n+1} + 1 < 0 \rightarrow \frac{n-2+n+1}{n+1} < 0 \rightarrow \frac{2n-1}{n+1} < 0 \rightarrow (-\frac{1}{2}, \frac{1}{2}) \quad (2)$

$(1) \cup (2) \rightarrow \left(-2, -\frac{1}{2} \right) \cup \left(-\frac{1}{2}, \frac{1}{2} \right)$

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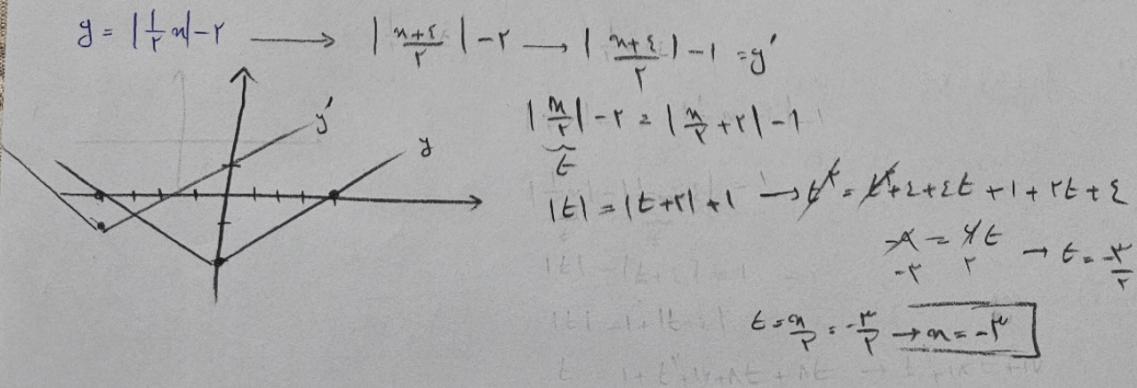
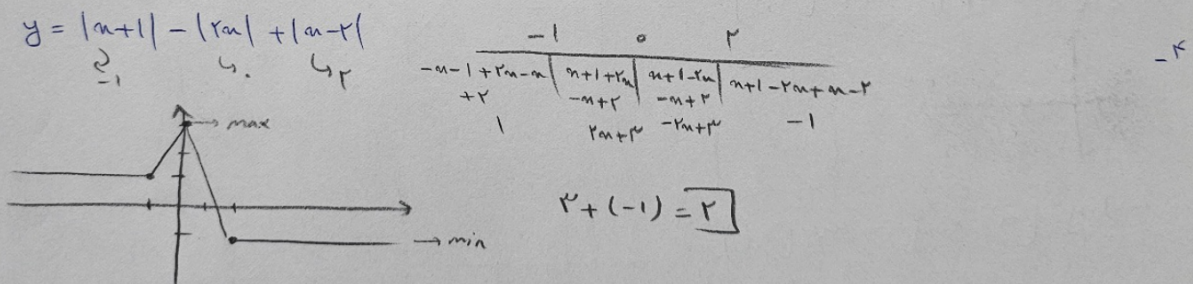
$n^2 < |n+4| \quad |n-\alpha| < \beta \rightarrow -\beta < n-\alpha < \beta \rightarrow \alpha-\beta < n < \alpha+\beta$

$n^2 - n - 4 < 0 \quad (n-2)(n+2) \quad \frac{-2 \quad 2}{+ \quad -} \quad (-2, 2)$

$\alpha + \beta = 2$
 $-\beta + \alpha = -2$
 $2\alpha = 1 \rightarrow \alpha = \frac{1}{2} \quad \beta = \frac{3}{2}$

$\alpha - \beta = -2 \rightarrow \alpha = \frac{1}{2} \quad \beta = \frac{3}{2}$

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$$f(x) = \left[(1-x) - \left[\frac{x+1}{2} \right] \right] \rightarrow \frac{-x}{2} \left[\frac{1-x}{2} \right] - \frac{1}{2} \rightarrow \frac{-x}{2} \left[\frac{1-x}{2} \right] - \frac{1}{2}$$

$$R_f = \left[-\frac{1}{2}, \frac{1}{2} \right]$$

ب) $f(x) = \frac{1}{x} - \left[\frac{x+1}{2} \right] \rightarrow \frac{1}{x} - \left[\frac{1}{2} \right] - 1 \rightarrow -1 < \frac{1}{x} < 1 \rightarrow R_f = [-1, 1]$

ج) $y = [-2x] - 1, x \in (-1, 1) \rightarrow -1 < x < 1 \rightarrow -2 < -2x < 2$

$-1 < x < \frac{1}{2}$

$-2 < -2x < -1 \rightarrow 1 < 2x < 2 \rightarrow [-2x] = 1 \rightarrow y = 0$

$\frac{1}{2} < x < \frac{3}{4}$

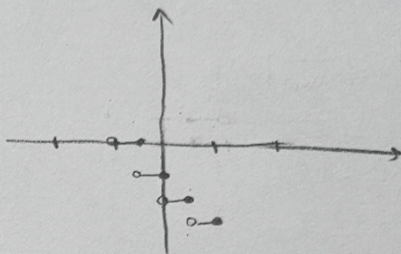
$-1 < -2x < -\frac{1}{2} \rightarrow [-2x] = 0 \rightarrow y = -1$

$\frac{3}{4} < x < \frac{5}{4}$

$-\frac{1}{2} < -2x < -1 \rightarrow [-2x] = -1 \rightarrow y = -2$

$\frac{5}{4} < x < \frac{3}{2}$

$-2 < -2x < -\frac{3}{2} \rightarrow [-2x] = -2 \rightarrow y = -3$



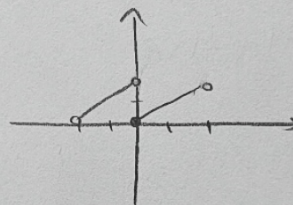
د) $y = x - 2 \left[\frac{x}{2} \right], x \in (-2, 2) \rightarrow -1 < \frac{x}{2} < 1$

$-2 < x < -1$

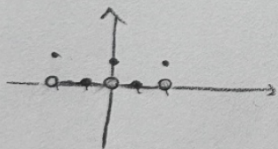
$-1 < \frac{x}{2} < 0 \rightarrow \left[\frac{x}{2} \right] = -1 \rightarrow y = x + 2$

$0 < x < 1$

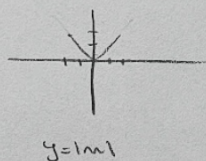
$0 < \frac{x}{2} < \frac{1}{2} \rightarrow \left[\frac{x}{2} \right] = 0 \rightarrow y = x$



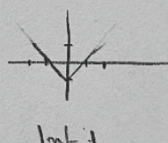
ه) $y = [|x| - 1], x \in [-2, 2]$



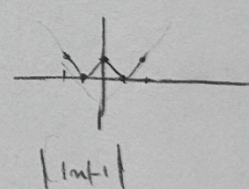
$[|x| - 1]$



$y = |x|$



$|x| - 1$

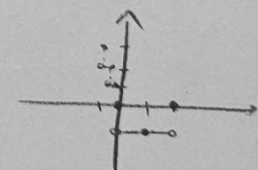
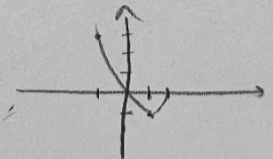


$[|x| - 1]$

و) $y = [\alpha x - \beta] , x \in [-1, 2]$

$\frac{-b}{\alpha} = \frac{1}{\alpha} = 1$

$1 - \alpha = -1$



$\alpha + [b] = 4, \quad b - [a] = 1, \quad \alpha + b = 9$

$[a] + [b] = 4$

$[b] - [a] = 1$

$\rightarrow [b] = 4 \rightarrow [b] = 3 \rightarrow b = [1, 4]$

$[a] = 1 \rightarrow a = [1, 2]$

$\rightarrow \alpha + b = [4, 9]$

$(1 + \sqrt{2})^4 + (1 - \sqrt{2})^4 = 194 \rightarrow (1 + \sqrt{2})^4 = 194 \rightarrow [(1 + \sqrt{2})^4] = 194$

$[(1 + \sqrt{2})^4]$