

$$x^2 + y^2 - 4x + 4y + k = 0$$

(۱)

$$k \leq 9$$

$$x^2 - 4x + k_1 \Rightarrow k_1 \leq 9$$

$$y^2 + 4y + k_2 \Rightarrow k_2 \leq 9$$

$$(\sqrt{x} - \sqrt{y})^2$$

$$(y + 4)^2 \Rightarrow k_2 \leq 9$$

$$x^2 - 4x + 4$$

$$y^2 + 4y + 4$$

5

(۲)

$$f(x) = \begin{cases} x^2 + x & ; x \geq a \\ x^2 + x + 1 & ; x < a \end{cases}$$

$$f(1) = 9$$

$$a > 0 \quad \text{شرط ذکر شده}$$

$$1 \rightarrow x^2 + x + 1 + 1 \leq 9$$

$$a \rightarrow a^2 + a = 3a + 1 \rightarrow a^2 + a - 2 = 0 \rightarrow (a+2)(a-1) = 0$$

$$f(1) = 0$$

$$a = 1 \quad \text{و چون } a > 0$$

(۳)

$$f(x) = \begin{cases} x^2 - b & ; |x+1| \geq 2 \\ x^2 + 4 & ; -2 \leq x \leq -1 \end{cases}$$

$$a + b \leq 9$$

$$\text{if } a = -2 \rightarrow 2(-2)^2 - b \leq a + 2(-2)$$

$$4 - b \leq -4$$

$$a + b \leq 9 \Rightarrow 9 \leq 9$$

20

(۴)

$$y = \sqrt{4x-a} + \sqrt{b-4x} \quad |4-a=0 \quad b=4$$

$$a \leq 4$$

$$b \leq 4$$

$$Dy = \{2\} \quad \frac{b}{a} \leq 9$$

$$\frac{b}{a} \leq \frac{4}{4} \Rightarrow \left(\frac{1}{4}\right)$$

الف) $y = \sqrt{\frac{n+1}{|n-k|}} + \sqrt{\frac{4-n}{n^2+kn+4}}$ $n=4$

Sign chart for n^2+kn+4 (roots at -1 and 4):

-1	4
+	-

Sign chart for $|n-k|$ (root at k):

k
-

Domain: $D_f = [-1, k) \cup (k, 4]$

ب) $y = \sqrt{[n]-4} + \sqrt{4-[n]}$

$[n] \geq 4 \Rightarrow n \geq 4$ $4-[n] \geq 0 \Rightarrow [n] \leq 4 \Rightarrow n \leq 4$

Sign chart for $n-4$ (root at 4):

4
-

Domain: $D_f = \emptyset$

ج) $y = \sqrt{\frac{n^2-n-4}{n^2-(k^2+k+4)}}$ $n=k$ $n=-k$

Factorization: $(n-k)(n+k)$ and $(n-k-4)(n-k+4)$

Sign chart for $n^2-(k^2+k+4)$ (roots at k and $-k$):

$-k$	k
+	-

Sign chart for $(n-k)(n+k)$ (roots at k and $-k$):

$-k$	k
+	-

Domain: $D_f = (-\infty, -k) \cup (k, +\infty)$

Sign chart for n^2-n-4 (roots at 1 and 4):

1	4
+	-

Sign chart for n^2+kn+4 (roots at -1 and 4):

-1	4
+	-

Domain: $D_f = (-\infty, -k) \cup (k, k) \cup (k, +\infty)$

د) $y = \sqrt{\frac{n^2-kn-4}{n^2+kn-4}}$ $n=1$ $n=k$ $n=-k$

Factorization: $(n-1)(n+k)(n-k)$ and $(n-k-4)(n-k+4)$

Sign chart for n^2-kn-4 (roots at 1 and 4):

1	4
+	-

Sign chart for n^2+kn-4 (roots at -1 and 4):

-1	4
+	-

Domain: $D_f = (-\infty, -k) \cup (k, k) \cup (k, +\infty)$

Long division for $\frac{n^2-kn-4}{n^2+kn-4}$:

n^2-kn-4	n^2+kn-4
$-kn+4n$	$-kn+4n$
$4n-4$	$4n-4$
$4n-4$	$4n-4$
0	0

Result: $1 + \frac{4n-4}{n^2+kn-4}$

Long division for $\frac{n^2-kn-4}{n^2+kn-4}$ (continued):

n^2-kn-4	n^2+kn-4
$-kn+4n$	$-kn+4n$
$4n-4$	$4n-4$
$4n-4$	$4n-4$
0	0

Result: $1 + \frac{4n-4}{n^2+kn-4}$

الف) $y = \sqrt{[-n] - r} \quad [-n] - r \geq 0$

$[-n] \geq r \Rightarrow -n \geq r$
 $n \leq -r$

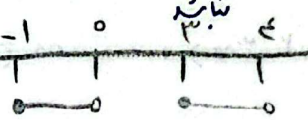
$D_f = (-\infty, -r]$

5 ب) $y = \frac{2n^2 + 4n}{[n] \leq r [n] - r} \quad ([n] - r)([n] + 1)$

$[n] \neq r \quad [n] \neq -1$

$[n] - r [n] - r \neq 0 \quad r \leq n < r+2 \quad -1 \leq n < 0$

$D_f = (-\infty, -1) \cup [0, r) \cup [r+2, +\infty)$

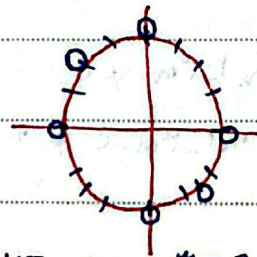


10

الف) $\frac{\cos n + 1}{\tan n + 1} \quad \frac{\cos}{\sin} \rightarrow \sin \neq 0$

$\tan n + 1$

$\tan \neq -1 \quad \cos \neq 0$



$\mathbb{R} - \left\{ \frac{k\pi}{r}, k\pi + \frac{r\pi}{r} \right\}$

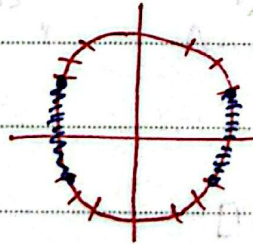
15

ب) $y = \sqrt{1 - \epsilon \sin^2 m} \quad 1 - \epsilon \sin^2 m \geq 0$

$1 \geq \epsilon \sin^2 m$

$\frac{1}{\epsilon} \geq \sin^2 m \rightarrow -\frac{1}{\sqrt{\epsilon}} \leq \sin m \leq \frac{1}{\sqrt{\epsilon}}$

$\left[r k \pi + \frac{0\pi}{4}, r k \pi + \frac{\sqrt{\pi}}{4} \right] \cup \left[r k \pi - \frac{\pi}{4}, r k \pi + \frac{\pi}{4} \right]$



$$\text{الف) } y = \sqrt{1 - \log_{\frac{1}{p}} n-1}$$

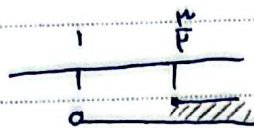
$$1 - \log_{\frac{1}{p}} n-1 > 0$$

$$\log_{\frac{1}{p}} n-1 \leq 1$$

(9)

$$n-1 > 0 \quad n > 1$$

$$n-1 > \frac{1}{p} \quad n > \frac{1}{p} + 1$$



$$D_f = \left[\frac{1}{p}, +\infty \right)$$

$$\text{ب) } y = \frac{\sqrt{n^r - n}}{1 - \log_{\frac{1}{p}} n^r - n}$$

$$n^r - n > 0$$

$$n(n-1) > 0$$

$$\frac{0}{+0-0+}$$

$$n^r - n > 0$$

$$n(n-1) > 0$$

$$\frac{0}{+0-0+}$$

$$D_f = (-\infty, 0) \cup (1, +\infty) - \{1\}$$

$$1 - \log_{\frac{1}{p}} n^r - n \neq 0$$

$$\log_{\frac{1}{p}} n^r - n \neq 1$$

$$n^r - n \neq 1$$

$$n \neq 1, -1$$

$$n^r - n - 1 \neq 0 \rightarrow (n-1)(n+1) \neq 0$$

$$\text{الف) } \sqrt{\frac{\epsilon n - n^r}{p - 1}}$$

$$\frac{\epsilon n - n^r}{p - 1} > 0$$

$$\frac{\epsilon n - n^r}{p - 1} > 0$$

$$\epsilon n - n^r > 0$$

$$\epsilon n^r - \epsilon n + n^r \leq 0$$

$$D_f = [1, p]$$

$$\frac{1}{+0-0+}$$

$$(n-1)(n-1) \leq 0$$

$$\text{ب) } y = \left(\frac{p_{m+0}}{p_{m+1}} \right)$$

$$\frac{p_{m+0}}{p_{m+1}} \in W$$

$$p_{m+0} \in p_{m+1}W + \epsilon W$$

$$p_{m+0} - p_{m+1}W \in \epsilon W - 0$$

$$n(p_{m+0}) \in \epsilon W - 0$$

$$D_f = \left\{ n \mid n = \frac{pk-0}{p-pk}, k \in W \right\}$$

$$n \in \frac{\epsilon W - 0}{p - pk}$$