

نام و نام خانوادگی بسیار زیاده. پاسخنامه تشریحی تکلیف شماره ۲... کلاس ... باید هیچ منبعی نداشته باشد...

$$2u^2 + y^2 - 2u + 2y + k = 0 \Rightarrow \underbrace{2u^2 - 2u + 1}_{2(u^2 - u + 1)} + \underbrace{y^2 + 2y + 1}_{(y+1)^2} = 0 \Rightarrow 2(u-1)^2 + (y+1)^2 = 0$$

$$k \Rightarrow 2 + 9 = 11 \rightarrow \text{جواب}$$

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$$f(u) = \begin{cases} u^2 + 2u & ; u \geq a \\ 2u + a + 2 & ; u \leq a \end{cases} \xrightarrow{\text{if } u=a} u^2 + 2u = 2u + a + 2 \Rightarrow a^2 + 2a = 2a + a + 2 \Rightarrow a^2 + 2a = 2a + 2$$

$$\Rightarrow a^2 + 2a - 2a - 2 = 0 \Rightarrow a^2 + a - 2 = 0 \Rightarrow (a-1)(a+2) = 0 \Rightarrow a = 1 \checkmark$$

چون a باید عدد صحیح باشد $a = -2 \rightarrow$ غلط

$$f(u) = \begin{cases} u^2 + 2u & ; u \geq 1 \\ 2u + 3 & ; u \leq 1 \end{cases} \rightarrow f(1) = \begin{cases} 1^2 + 2 \times 1 = 3 \\ 2 \times 1 + 3 = 5 \end{cases} = 5 = f(1) \rightarrow \text{جواب}$$

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$$f(u) = \begin{cases} 2u^2 - b & ; |u+1| \geq 2 \\ a + 2u & ; -3 \leq u \leq -1 \end{cases} \Rightarrow f(u) = \begin{cases} 2u^2 - b & ; u \geq 1, u \leq -3 \\ a + 2u & ; -3 \leq u \leq -1 \end{cases}$$

$$\star |u+1| \geq 2 \rightarrow u+1 \geq 2 \rightarrow u \geq 1$$

$$u+1 \leq -2 \rightarrow u \leq -3$$

$$\text{if } u = -3 \quad 2u^2 - b = a + 2u \Rightarrow 2 \times (-3)^2 - b = a + 2 \times (-3) \Rightarrow 2 \times 9 - b = a - 6 \Rightarrow 18 - b = a - 6$$

$$\Rightarrow 18 + 6 = a + b \Rightarrow a + b = 24 \rightarrow \text{جواب}$$

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$$y = \sqrt{4u-a} + \sqrt{b-3u} \rightarrow 4u-a \geq 0 \rightarrow 4u \geq a \rightarrow u \geq \frac{a}{4}$$

$$\rightarrow b-3u \geq 0 \rightarrow -3u \geq -b \rightarrow u \leq \frac{b}{3}$$

$$\Rightarrow \frac{a}{4} \leq \frac{b}{3} \Rightarrow \frac{a}{b} \leq \frac{4}{3} \Rightarrow \frac{a}{b} = \frac{4}{3} = \frac{1}{\frac{3}{4}} \Rightarrow \frac{a}{b} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

$$\Rightarrow a = 12, b = 4 \Rightarrow \frac{b}{a} = \frac{4}{12} = \frac{1}{3} \rightarrow \text{جواب}$$

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$$\text{الف) } y = \sqrt{\frac{u+1}{|u-3|}} + \sqrt{\frac{4-u}{u^2+2u+2}} \Rightarrow \frac{u+1}{|u-3|} \geq 0 \text{ و } \frac{4-u}{u^2+2u+2} \geq 0$$

$$I \cap II \rightarrow D_f = [-1, 3) \cup (3, 4]$$

جواب

$$\star \rightarrow \frac{u+1}{|u-3|} \geq 0 \rightarrow \begin{matrix} u = -1 \\ u = 3 \end{matrix} \Rightarrow \frac{-1}{-4+9} \Rightarrow [-1, 3) \cup (3, +\infty) = D_f(I)$$

$$\star \rightarrow \frac{4-u}{u^2+2u+2} \geq 0 \rightarrow 4-u \geq 0 \Rightarrow u \leq 4 = D_f(II)$$

$$\text{ب) } y = \sqrt{[u]-2} + \sqrt{2-[u]} \Rightarrow [u]-2 \geq 0 \text{ و } 2-[u] \geq 0$$

$$\star \rightarrow [u]-2 \geq 0 \rightarrow [u] \geq 2 \rightarrow u \geq 2 \rightarrow [2, +\infty) = D_f(I)$$

$$\star \rightarrow 2-[u] \geq 0 \rightarrow [u] \leq 2 \rightarrow u \leq 3 \rightarrow (-\infty, 3) = D_f(II)$$

$$I \cap II \rightarrow D_f = \emptyset \rightarrow \text{جواب}$$

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پس هیچ حقیقی ندارد

$$\text{الف) } y = \sqrt{\frac{u^2 - u - 4}{u^2 - 13u^2 + 34}} \rightarrow \frac{u^2 - u - 4}{u^2 - 13u^2 + 34} \geq 0 \Rightarrow$$

$$\star \rightarrow u^2 - 13u^2 + 34 \Rightarrow u^2 = t \Rightarrow t^2 - 13t + 34 = 0 \Rightarrow (t-2)(t-9) = 0 \rightarrow t=2 \rightarrow u^2=2 \rightarrow u=\pm\sqrt{2}$$

$$\star \rightarrow u^2 - u - 4 = 0 \Rightarrow (u-3)(u+2) = 0 \Rightarrow u=3 \rightarrow t=9 \rightarrow u^2=9 \rightarrow u=\pm 3$$

$$\frac{-3 \quad -2 \quad 2 \quad 3}{+\quad -\quad -\quad +} \rightarrow (-\infty, -3) \cup (2, 3) \cup (3, +\infty) = D_f \rightarrow \text{جواب}$$

$$\text{ب) } y = \sqrt{\frac{u^3 - 2u^2 - 5u + 4}{u^3 + 2u^2 - 5u - 4}} \Rightarrow \frac{u^3 - 2u^2 - 5u + 4}{u^3 + 2u^2 - 5u - 4} \geq 0$$

$$\star \rightarrow \begin{array}{r} u^3 - 2u^2 - 5u + 4 \\ -(u^3 + 2u^2) \\ \hline -4u^2 - 5u + 4 \\ -(-4u^2 + 4u) \\ \hline -9u + 4 \\ -(-9u + 9) \\ \hline -5 \end{array} \quad \begin{array}{r} u-1 \\ u^2-u-4 \\ \hline u^2-u-4 \\ -(u^2-u-4) \\ \hline 0 \end{array}$$

$$\begin{aligned} & \rightarrow (u-1)(u^2-u-4) = 0 \\ & (u-1)(u+2)(u-3) = 0 \\ & \rightarrow u=1 \quad \rightarrow u=-2 \quad \rightarrow u=3 \end{aligned}$$

$$\star \rightarrow \begin{array}{r} u^3 + 2u^2 - 5u - 4 \\ -(u^3 + u^2) \\ \hline u^2 - 5u - 4 \\ -(u^2 + u) \\ \hline -6u - 4 \\ -(-6u - 3) \\ \hline -1 \end{array} \quad \begin{array}{r} u+1 \\ u^2+u-4 \\ \hline u^2+u-4 \\ -(u^2+u-4) \\ \hline 0 \end{array}$$

$$\begin{aligned} & \rightarrow (u+1)(u^2+u-4) = 0 \\ & (u+1)(u-2)(u+3) = 0 \\ & \rightarrow u=-1 \quad \rightarrow u=2 \quad \rightarrow u=-3 \end{aligned}$$

$$\Rightarrow \frac{-3 \quad -2 \quad -1 \quad 1 \quad 2 \quad 3}{+\quad -\quad -\quad -\quad +\quad +} \Rightarrow D_f = (-\infty, -3) \cup [-2, -1) \cup [1, 2) \cup [3, +\infty)$$

لکه مجموع ضرایب توانهای فرد برابر ۰ شود پس عبارت در $u+1$ بخش پذیر است

جواب

$$\text{الف) } y = \sqrt{[-u]-2} \Rightarrow [-u]-2 \geq 0 \rightarrow [-u] \geq 2 \rightarrow u \leq -2 \rightarrow D_f = (-\infty, -2] \rightarrow \text{جواب}$$

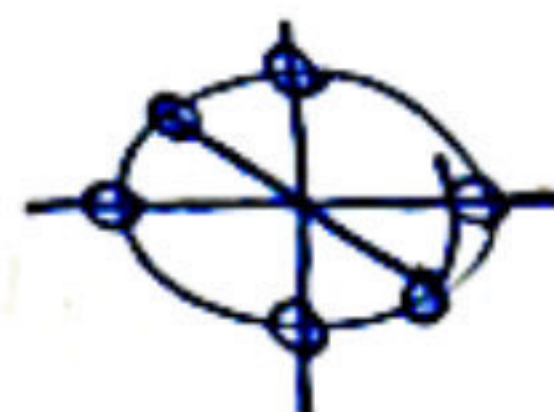
$$\text{ب) } y = \frac{5u^2+4}{[u]^2-2[u]-3} \rightarrow \begin{aligned} & \star \text{ صورت همواره مثبت است چون } 5u^2+4 > 0 \\ & [u]^2-2[u]-3 \neq 0 \rightarrow [u] \neq 3 \rightarrow u \notin [3, 4) \\ & \rightarrow [u] \neq -1 \rightarrow u \notin [-1, 0) \end{aligned}$$

$$D_f = R - \{[3, 4) \cup [-1, 0)\} \rightarrow \text{جواب}$$

$$\text{الف) } y = \frac{\cot u + 1}{\tan u + 1} \rightarrow \cot u = \frac{\cos u}{\sin u} \rightarrow \sin u \neq 0, \tan u = \frac{\sin u}{\cos u} \rightarrow \cos u \neq 0 \Rightarrow u \neq \frac{\pi}{2}, u \neq \frac{3\pi}{2}, u \neq \frac{\pi}{2}, u \neq \frac{3\pi}{2}$$

$$\star \rightarrow \tan u + 1 \neq 0 \rightarrow \tan u \neq -1$$

$$u \neq \frac{3\pi}{4}, u \neq \frac{7\pi}{4}$$



$$\rightarrow R - \left\{ \frac{3\pi}{4}, \frac{7\pi}{4} \right\} \Rightarrow D_f \rightarrow \text{جواب}$$

$$\text{ب) } y = \sqrt{1 - 5\sin^2 u} \rightarrow 1 - 5\sin^2 u \geq 0 \rightarrow 5\sin^2 u \leq 1 \rightarrow \sin^2 u \leq \frac{1}{5} \rightarrow -\frac{1}{\sqrt{5}} \leq \sin u \leq \frac{1}{\sqrt{5}}$$



$$\rightarrow D_f = \left[k\pi - \frac{\pi}{5}, k\pi + \frac{\pi}{5} \right] \Rightarrow \text{جواب}$$

$$\text{الف) } y = \sqrt{1 - \log_{\frac{1}{2}} u - 1} \rightarrow 1 - \log_{\frac{1}{2}} u - 1 \geq 0 \rightarrow \log_{\frac{1}{2}} u \leq 1 \rightarrow u - 1 \geq \frac{1}{2} \rightarrow u \geq \frac{3}{2} \quad I$$

$$\log_{\frac{1}{2}} u \rightarrow \begin{cases} \frac{1}{2} > 0 \\ \frac{1}{2} \neq 1 \\ u - 1 > 0 \rightarrow u > 1 \rightarrow II \end{cases}$$

کوینک از منفی پس جفت جفت کرد

$$I \cap II \Rightarrow D_f = \left[\frac{3}{2}, +\infty \right) \rightarrow \text{جواب}$$

$$\text{ب) } y = \frac{\sqrt{u^2 - u}}{1 - \log_{\frac{1}{2}} u - 2u} \rightarrow u^2 - u \geq 0, 1 - \log_{\frac{1}{2}} u - 2u \neq 0$$

$$\star \rightarrow u(u-1) \geq 0 \rightarrow \begin{aligned} & u=0 \rightarrow \frac{0}{+} \rightarrow (-\infty, 0] \cup [1, +\infty) \quad I \\ & u=1 \rightarrow \frac{0}{+} \rightarrow (-\infty, 0] \cup [1, +\infty) \quad I \end{aligned}$$

$$\star \rightarrow 1 - \log_{\frac{1}{2}} u - 2u \neq 0 \rightarrow \log_{\frac{1}{2}} u - 2u \neq 1 \rightarrow u^2 - 2u \neq 4 \rightarrow u^2 - 2u - 4 \neq 0 \rightarrow (u+1)(u-5) \neq 0 \rightarrow u \neq -1, u \neq 5 \quad II$$

$$\log_{\frac{1}{2}} u - 2u \rightarrow \begin{cases} u^2 - 2u > 0 \rightarrow u(u-2) > 0 \rightarrow \frac{0}{+} \rightarrow (-\infty, 0) \cup (2, +\infty) \quad III \\ \frac{1}{2} \neq 1 \\ \frac{1}{2} > 0 \end{cases}$$

$$I \cap II \cap III \rightarrow D_f = (-\infty, 0) \cup (2, +\infty) - \{-1, 5\} \Rightarrow \text{جواب}$$

$$\text{الف) } y = \sqrt{2u - u^2} - 1 \rightarrow 2u - u^2 - 1 \geq 0 \rightarrow 2u - u^2 \geq 1 \rightarrow 2u - u^2 \geq 1 \rightarrow 2u - u^2 \geq 1$$

$$\Rightarrow u^2 - 2u + 1 \leq 0 \rightarrow (u-1)^2 \leq 0 \rightarrow \frac{1}{+} \rightarrow D_f = [1, 1] \Rightarrow \text{جواب}$$

$$\text{ب) } y = \left(\frac{2u+5}{2u+2} \right)! \rightarrow \frac{2u+5}{2u+2} \in \mathbb{N} \rightarrow u \in \frac{-5-2k}{2k-2} \rightarrow D_f = \left\{ u \mid u = \frac{-5-2k}{2k-2}, k \in \mathbb{N} \right\}$$

جواب