

$$2x^2 + y^2 - 4x + 4y + k = 0 \Rightarrow \underbrace{2x^2 - 4x + 2}_{2(x^2 - 2x + 1)} + \underbrace{y^2 + 4y + 4}_{(y+2)^2} = 0$$

$$k = 2 + 4 = 11$$

$$= 2x(x-1)^2$$

$$f(x) = \begin{cases} x^2 + 4x & x \geq a \\ 2x + a + 2 & x \leq a \end{cases} \xrightarrow{x=a} a^2 + 4a = 2a + a + 2$$

$$a^2 + a - 2 = 0 \quad (a+2)(a-1) = 0$$

$$x=1 \rightarrow x^2 + 4x = 1 + 4 = 5$$

$$\left\{ \begin{array}{l} a = -2 \\ a = 1 \end{array} \right. \leftarrow \text{برای هر دو حالت } a \text{، یک جواب دارد } a \text{ است.}$$

$$f(x) = \begin{cases} 2x^2 - b & |x+1| \geq 2 \Rightarrow x+1 \geq 2 \Rightarrow x \geq 1 \\ a + 2x & -3 \leq x \leq -1 \end{cases}$$

$$x+1 \leq -2 \Rightarrow x \leq -3$$

$$x = -3 \rightarrow 11 - b = a - 4 \Rightarrow a + b = 24$$

$$\left. \begin{array}{l} 4x - a \geq 0 \quad x \geq \frac{a}{4} \\ b - 3x \geq 0 \quad x \leq \frac{b}{3} \end{array} \right\} \Rightarrow \frac{a}{4} = \frac{b}{3} = 2 \Rightarrow \begin{array}{l} a = 8 \\ b = 6 \end{array} \Rightarrow \frac{b}{a} = \frac{6}{8} = \frac{3}{4}$$

$$\text{الف)} \quad y = \sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{4-x}{x^2+2x+4}}$$

$$\frac{x+1}{|x-3|} \geq 0 \quad \frac{-1}{-1} + \frac{4}{0} + \frac{4}{1} = 7$$

$$\frac{4-x}{x^2+2x+4} \geq 0 \Rightarrow 4-x \geq 0 \Rightarrow x \leq 4 \quad (-\infty, 4] \quad (2) \quad [1, +\infty) - \{3\} \quad (1)$$

$$\Delta = 4 - 16 = -12 < 0 \Rightarrow \text{مخرج همواره مثبت است}$$

$$(1) \cap (2) \rightarrow D_f = [1, 4] - \{3\}$$

$$\text{ب)} \quad y = \sqrt{[x]-4} + \sqrt{2-[x]} \quad [x]-4 \geq 0 \Rightarrow [x] \geq 4 \Rightarrow [4, +\infty) \quad (1)$$

$$2-[x] \geq 0 \Rightarrow [x] \leq 2 \Rightarrow (-\infty, 2] \quad (2)$$

$$(1) \cap (2) \rightarrow D_f = \emptyset$$

$$\text{الف)} \quad y = \sqrt{\frac{x^2-x-4}{x^2-13x^2+34}}$$

$$\frac{x^2-x-4}{x^2-13x^2+34} \geq 0 \quad (3) \quad (-2)$$

$$\frac{-4}{-4} \quad \frac{-1}{-1} \quad \frac{4}{4} \quad \frac{34}{34}$$

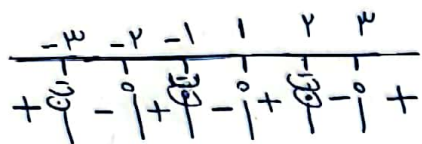
$$\rightarrow (x^2-4)(x^2-9)$$

$$\begin{array}{cc} (\pm 2) & (\pm 3) \end{array}$$

$$D_f = (-\infty, -3) \cup (2, 3) \cup (3, +\infty)$$

ب) $y = \sqrt{\frac{x^3 - 2x^2 - 5x + 4}{x^3 + 2x^2 - 5x - 4}}$

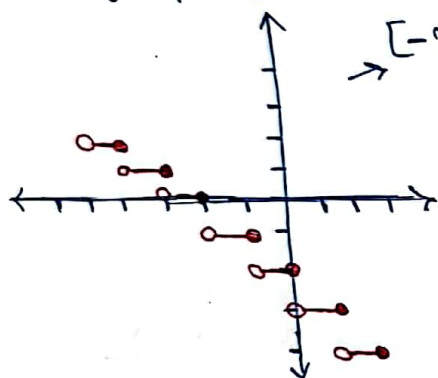
$(x-1)(x-3)(x+2)$ y_0
 $(x+1)(x+3)(x-2)$
 $-1 \quad -3 \quad 2$



$$\begin{array}{r|l} x^3 - 2x^2 - 5x + 4 & x-1 \\ \hline -x^3 + x^2 & \\ \hline -x^2 - 5x + 4 & \\ + x^2 - x & \\ \hline -4x + 4 & \\ -4x + 4 & \\ \hline 0 & \end{array} \quad \begin{array}{r|l} x^3 + 2x^2 - 5x - 4 & x+1 \\ \hline -x^3 - x^2 & \\ \hline x^2 - 5x - 4 & \\ -x^2 - x & \\ \hline -4x - 4 & \\ +4x + 4 & \\ \hline 0 & \end{array}$$

$D_f = (-\infty, -3) \cup [-2, -1) \cup [1, 2) \cup [3, +\infty)$

الف) $y = \sqrt{[-x] - 2}$

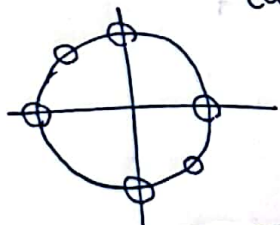


$[-x] - 2 \geq 0$
 $\Rightarrow x \leq -2$
 $D_f = (-\infty, -2]$

ب) $y = \frac{5x^2 + 4}{[x]^2 - 2[x] - 3}$

$[x]^2 - 2[x] - 3 \neq 0$
 $([x] - 3)([x] + 1) \neq 0$
 $[x] \neq 3 \Rightarrow x \notin [3, \infty)$
 $[x] \neq -1 \Rightarrow x \notin [-1, 0)$
 $D_f = \mathbb{R} - [3, \infty) - [-1, 0)$

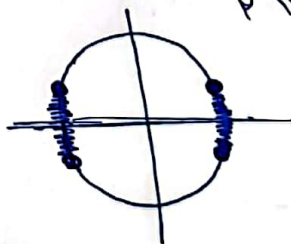
الف) $y = \frac{\cot x + 1}{\tan x + 1}$ $\tan x + 1 \neq 0 \Rightarrow \tan x \neq -1$



$D_f = \mathbb{R} - \left\{ \frac{k\pi}{4}, k\pi - \frac{\pi}{4} \right\}$

ب) $y = \sqrt{1 - \kappa \sin^2 x}$

$1 - \kappa \sin^2 x \geq 0$
 $\kappa \sin^2 x \leq 1 \Rightarrow \sin^2 x \leq \frac{1}{\kappa}$
 $\Rightarrow -\frac{1}{\sqrt{\kappa}} \leq \sin x \leq \frac{1}{\sqrt{\kappa}}$



$D_f = \left[k\pi - \frac{\pi}{4}, k\pi + \frac{\pi}{4} \right]$

الف) $y = \sqrt{1 - \log_{\frac{1}{r}} x - 1}$ $x - 1 \geq 0 \Rightarrow x \geq 1$ ①

$1 - \log_{\frac{1}{r}} x - 1 \geq 0 \Rightarrow \log_{\frac{1}{r}} x \leq 1 \Rightarrow$

$x - 1 \geq \frac{1}{r} \Rightarrow x \geq \frac{r+1}{r}$ ② $\Rightarrow D_f = \left[\frac{r+1}{r}, +\infty \right)$

ب) $y = \frac{\sqrt{x^2 - x}}{1 - \log_{\kappa} x^2 - 3x}$

$x^2 - x \geq 0 \Rightarrow x(x-1) \geq 0$
 $1 - \log_{\kappa} x^2 - 3x \neq 0 \Rightarrow \log_{\kappa} x^2 - 3x \neq 1$
 $x^2 - 3x - \varepsilon \neq 0$

$\Rightarrow x^2 - 3x - \varepsilon \neq 0$ $(x - \varepsilon)(x + 1) \neq 0$ $x \neq \kappa, x \neq -1$ ③

$x^2 - 3x \geq 0$ $x(x-3) \geq 0$ $\frac{0 \quad 3}{+ \quad - \quad +}$ $(-\infty, 0) \cup (3, +\infty)$ ④

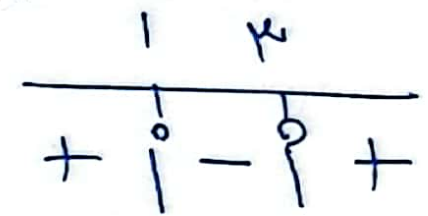
$$\textcircled{1} \wedge \textcircled{2} \wedge \textcircled{3} \rightarrow D_f = (-\infty, 0) \cup (1, +\infty) - \{1, -1\}$$

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الف) $y = \sqrt{1 - x^2} \Rightarrow y^2 = 1 - x^2 \Rightarrow x^2 + y^2 = 1$

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$$\Rightarrow -x^2 + \varepsilon x \geq 0 \Rightarrow x^2 - \varepsilon x \leq 0 \Rightarrow (x-1)(x-\varepsilon) \leq 0$$



$$D_f = [1, \varepsilon]$$

ب) $y = \left(\frac{wx + \omega}{1x + \varepsilon} \right)!$ $\frac{wx + \omega}{1x + \varepsilon} \in \omega \Rightarrow x \in \frac{-\varepsilon\omega + \omega}{1\omega - \omega}$

$$D_f = \left\{ x \mid x = \frac{\omega - \varepsilon k}{1k - \omega}, k \in \omega \right\}$$