

$$2x^2 + y^2 - 4x + 4y + k = 0 \rightarrow 2x^2 - 4x + y^2 + 4y + k = 0 \rightarrow 2(x^2 - 2x) + y^2 + 4y + k = 0$$

$$\rightarrow 2(x-1)^2 + (y+2)^2 = 0 \rightarrow k = 4 + 2 = 11$$

مسئله (۱) $k = ?$

$$f(x) = \begin{cases} x^2 + 4x & x > a \\ 2x + a + 2 & x \leq a \end{cases}$$

$$x=a \rightarrow a^2 + 4a = 2a + a + 2 \rightarrow a(a+4) = 3a+2$$

$$\rightarrow a^2 + 4a - 3a - 2 = 0 \rightarrow a^2 + a - 2 = 0 \rightarrow (a+2)(a-1) = 0$$

مسئله (۲) $f(1) = ?$

$$\rightarrow f(x) = \begin{cases} x^2 + 4x & x > 1 \\ 2x + 3 & x \leq 1 \end{cases}$$

$$f(1) = 1 + 4 = 5$$

$$\rightarrow \begin{cases} a = -2 \\ a = 1 \end{cases} \checkmark$$

$$f(x) = \begin{cases} 2x^2 - b & |x+1| > 2 \\ a + 2x & -3 \leq x \leq -1 \end{cases}$$

$$x = -3 \rightarrow 18 - b = a - 4 \rightarrow a + b = 22$$

مسئله (۳) $a + b = ?$

$$y = \sqrt{4x-a} + \sqrt{b-3x} \quad D_y = \{x\}$$

$$4x-a \xrightarrow{x=2} 4x-a=0 \rightarrow 4x=a \rightarrow a=12$$

$$b-3x \xrightarrow{x=2} b-3x=0 \rightarrow b=3x \rightarrow b=6$$

$$\frac{b}{a} = \frac{6}{12} = \frac{1}{2}$$

مسئله (۴) $\frac{b}{a} = ?$

$$y = \sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{4-x}{x^2+2x+5}} \rightarrow D: (-\infty, 4]$$

مسئله (۵) $D_f = ?$

$$\frac{-1}{-3} + \frac{1}{3} \rightarrow D: [-1, +\infty) - \{3\}$$

$$\Rightarrow D_f = [-1, 4] - \{3\}$$

$$y = \sqrt{[x]-4} + \sqrt{2-[x]}$$

$$[x]-4 \geq 0 \rightarrow [x] \geq 4 \rightarrow D: [4, +\infty)$$

$$\Rightarrow D_f = \emptyset$$

$$2-[x] \geq 0 \rightarrow 2 \geq [x] \rightarrow D: (-\infty, 3)$$

$$y = \sqrt{\frac{x^2-x-4}{x^2-13x^2+34}} = \sqrt{\frac{(x-3)(x+2)}{(x+2)(x-2)(x-3)(x+3)}}$$

مسئله (۶) $D_f = ?$

$$\Rightarrow D_f = (-\infty, -3) \cup (2, 3) \cup (3, +\infty)$$

$$y = \sqrt{\frac{x^3-2x^2-5x+4}{x^3+2x^2-5x-4}} = \sqrt{\frac{(x-1)(x-3)(x+2)}{(x+1)(x+3)(x-2)}}$$

$$\Rightarrow D_f = (-\infty, -3) \cup [-2, -1) \cup [1, 2) \cup [3, +\infty)$$

$$y = \sqrt{[-x]-2} \quad [-x]-2 \geq 0 \rightarrow [-x] \geq 2 \Rightarrow D_f = (-\infty, -2]$$

مسئله (۷) $D_f = ?$

$$y = \frac{5x^2+4}{[x]^2-2[x]-3} \rightarrow ([x]-3)([x]+1) \Rightarrow D_f = \mathbb{R} - \{[3, 4) \cup [-1, 0)\}$$

$$y = \frac{\cos x + 1}{\tan x + 1} \quad \tan x + 1 \neq 0 \rightarrow \tan x \neq -1 \Rightarrow D_f = \mathbb{R} - \{\frac{k\pi}{2}, k\pi - \frac{\pi}{4}\}$$

مسئله (۸) $D_f = ?$

$$y = \sqrt{1-\sin^2 x} \quad 1-\sin^2 x \geq 0 \rightarrow 1 \geq \sin^2 x \rightarrow \frac{1}{2} \geq \sin^2 x \rightarrow -\frac{1}{2} \leq \sin x \leq \frac{1}{2}$$



$$\Rightarrow D_f = [2k\pi - \frac{\pi}{2}, 2k\pi + \frac{\pi}{2}] \cup [2k\pi + \frac{3\pi}{4}, 2k\pi + \frac{5\pi}{4}]$$

الف) $y = \sqrt{1 - \log_{\frac{1}{r}} x - 1}$

$$1 - \log_{\frac{1}{r}} x - 1 \geq 0 \rightarrow 1 \geq \log_{\frac{1}{r}} x - 1 \rightarrow \frac{1}{r} \leq x - 1 \rightarrow x \geq \frac{r}{r} \\ x - 1 \neq 0 \rightarrow x \neq 1$$

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$$\Rightarrow D_f = \left[\frac{r}{r}, +\infty \right)$$

ب) $y = \frac{\sqrt{x^r - r}}{1 - \log_{\frac{1}{r}} x^r - r}$ $\rightarrow x(x-1) \geq 0$ $\frac{+}{+} \frac{+}{-} \frac{-}{+} \frac{-}{-} \rightarrow \mathbb{R} - (0, 1)$

$1 - \log_{\frac{1}{r}} x^r - r \neq 0$

$$\begin{cases} x^r - r \neq 0 \rightarrow (x-r)(x+r) \rightarrow x \neq r, -1 \\ x^r - r > 0 \rightarrow x(x-r) > 0 \rightarrow \frac{+}{+} \frac{+}{-} \frac{-}{+} \frac{-}{-} \rightarrow \mathbb{R} - [0, r] \end{cases}$$

$$\Rightarrow D_f = (-\infty, 0) \cup (r, +\infty) - \{r, -1\}$$

الف) $y = \sqrt{r^r x - x^r} - 1$

$$r^r x - x^r - 1 \geq 0 \rightarrow r^r x - x^r \geq 1 \rightarrow r^r x - x^r \geq r^r \rightarrow r^r x - x^r \geq r^r \rightarrow \\ x^r - r^r x + r^r \leq 0 \rightarrow \frac{+}{+} \frac{+}{-} \frac{-}{+} \frac{-}{-} \Rightarrow D_f = [1, r]$$

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ب) $y = \left(\frac{r^n + \omega}{r^n + r} \right)!$ $\rightarrow \left\{ n = \frac{\omega - r^k}{r^k - r} \mid k \in \mathbb{W} \right\}$

$$\frac{r^n + \omega}{r^n + r} \in \mathbb{W} \rightarrow n \in \frac{\omega - r^k}{r^k - r}$$