

$$2x^2 + y^2 - 4x + 4y + k = 0 \rightarrow 2x^2 - 4x + y^2 + 4y + k = 0 \rightarrow 2(x^2 - 2x) + y^2 + 4y + k = 0$$

$$\rightarrow 2(x-1)^2 + (y+2)^2 = 0 \rightarrow k = 4 + 2 = 11$$

مسئله ۱) $k = ?$

$$f(x) = \begin{cases} x^2 + 4x & x > a \\ 2x + a + 2 & x \leq a \end{cases}$$

$$x=a \rightarrow a^2 + 4a = 2a + a + 2 \rightarrow a(a+4) = 3a+2$$

$$\rightarrow a^2 + 4a - 3a - 2 = 0 \rightarrow a^2 + a - 2 = 0 \rightarrow (a+2)(a-1) = 0$$

$$\rightarrow \begin{cases} a = -2 \\ a = 1 \end{cases}$$

مسئله ۲) $f(1) = ?$

$$\rightarrow f(x) = \begin{cases} x^2 + 4x & x > 1 \\ 2x + 3 & x \leq 1 \end{cases}$$

$$f(1) = 1 + 4 = 5$$

$$\rightarrow \begin{cases} a = -2 \\ a = 1 \end{cases}$$

$$f(x) = \begin{cases} 2x^2 - b & |x+1| > 2 \\ a + 2x & -3 \leq x \leq -1 \end{cases}$$

$$x = -3 \rightarrow 18 - b = a - 4 \rightarrow a + b = 22$$

مسئله ۳) $a + b = ?$

$$y = \sqrt{4x-a} + \sqrt{b-3x} \quad D_y = \{2\}$$

$$4x-a \xrightarrow{x=2} 4x-a=0 \rightarrow 4x=a \rightarrow a=12$$

$$b-3x \xrightarrow{x=2} b-3x=0 \rightarrow b=3x \rightarrow b=6$$

$$\frac{b}{a} = \frac{6}{12} = \frac{1}{2}$$

مسئله ۴) $\frac{b}{a} = ?$

$$y = \sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{4-x}{x^2+2x+5}} \rightarrow D_f = (-\infty, 4]$$

مسئله ۵) $D_f = ?$

$$\frac{-1}{-3} + \frac{4}{5} \rightarrow D = [-1, +\infty) - \{3\}$$

$$\Rightarrow D_f = [-1, 4] - \{3\}$$

$$y = \sqrt{[x]-4} + \sqrt{2-[x]}$$

$$[x]-4 \geq 0 \rightarrow [x] \geq 4 \rightarrow D: [4, +\infty)$$

$$2-[x] \geq 0 \rightarrow 2 \geq [x] \rightarrow D: (-\infty, 3)$$

$$\Rightarrow D_f = \emptyset$$

$$y = \sqrt{\frac{x^2-x-4}{x^2-13x^2+34}} = \sqrt{\frac{(x-3)(x+2)}{(x+2)(x-2)(x-3)(x+3)}}$$

$$\begin{array}{c|ccc} -3 & -2 & 2 & 3 \\ \hline + & - & - & + \end{array}$$

$$\Rightarrow D_f = (-\infty, -3) \cup (2, 3) \cup (3, +\infty)$$

$$y = \sqrt{\frac{x^3-2x^2-5x+4}{x^3+2x^2-5x-4}} = \sqrt{\frac{(x-1)(x-3)(x+2)}{(x+1)(x+3)(x-2)}}$$

$$\begin{array}{c|cccc} -3 & -2 & -1 & 1 & 2 & 3 \\ \hline + & - & + & - & + & + \end{array}$$

$$\Rightarrow D_f = (-\infty, -3) \cup [-2, -1) \cup [1, 2) \cup [3, +\infty)$$

$$y = \sqrt{[-x]-2}$$

$$[-x]-2 \geq 0 \rightarrow [-x] \geq 2 \Rightarrow D_f = (-\infty, -2]$$

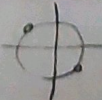
مسئله ۷) $D_f = ?$

$$y = \frac{5x^2+4}{[x]^2-2[x]-3} \rightarrow ([x]-3)([x]+1)$$

$$\Rightarrow D_f = \mathbb{R} - \{[3, 4) \cup [-1, 0)\}$$

$$y = \frac{\cos x + 1}{\tan x + 1}$$

$$\tan x + 1 \neq 0 \rightarrow \tan x \neq -1$$

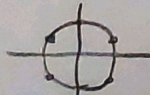


$$\Rightarrow D_f = \mathbb{R} - \{\frac{3\pi}{4}, \frac{5\pi}{4}\}$$

مسئله ۸) $D_f = ?$

$$y = \sqrt{1-f \sin^2 x}$$

$$1-f \sin^2 x \geq 0 \rightarrow 1 \geq f \sin^2 x \rightarrow \frac{1}{f} \geq \sin^2 x \rightarrow -\frac{1}{\sqrt{f}} \leq \sin x \leq \frac{1}{\sqrt{f}}$$



$$\Rightarrow D_f = [2k\pi - \frac{\pi}{\sqrt{f}}, 2k\pi + \frac{\pi}{\sqrt{f}}] \cup [2k\pi + \frac{3\pi}{\sqrt{f}}, 2k\pi + \frac{5\pi}{\sqrt{f}}]$$

الف) $y = \sqrt{1 - \log_{\frac{1}{r}} x - 1}$

$1 - \log_{\frac{1}{r}} x - 1 \geq 0 \rightarrow 1 \geq \log_{\frac{1}{r}} x - 1 \rightarrow \frac{1}{r} \leq x - 1 \rightarrow x \geq \frac{r}{r} = 1$
 $x - 1 \neq 0 \rightarrow x \neq 1$

$\Rightarrow D_f = \left[\frac{r}{r}, +\infty\right)$

ب) $y = \frac{\sqrt{x^r - r}}{1 - \log_{\frac{1}{r}} x^r - r}$ $\rightarrow x(x-1) \geq 0$ $\frac{+}{+} \frac{+}{-} \frac{-}{+} \frac{-}{-} \rightarrow \mathbb{R} - (0, 1)$

$1 - \log_{\frac{1}{r}} x^r - r \neq 0$

$\begin{cases} x^r - r \neq 0 \rightarrow (x-r)(x+1) \rightarrow x \neq r, -1 \\ x^r - r > 0 \rightarrow x(x-r) > 0 \rightarrow \frac{+}{+} \frac{+}{-} \frac{-}{+} \frac{-}{-} \rightarrow \mathbb{R} - [0, r] \end{cases}$

$\Rightarrow D_f = (-\infty, 0) \cup (r, +\infty) - \{r, -1\}$

الف) $y = \sqrt{r^x - x^r} - 1$

$r^x - x^r - 1 \geq 0 \rightarrow r^x - x^r \geq 1$ $\rightarrow r^x \geq x^r + 1$ $\rightarrow r^x \geq x^r$ $\rightarrow x \geq r$

$x^r - x^r + r \leq 0 \rightarrow \frac{+}{+} \frac{+}{-} \frac{-}{+} \frac{-}{-} \rightarrow \frac{+}{+} \frac{+}{-} \frac{-}{+} \frac{-}{-} \Rightarrow D_f = [1, r]$

ب) $y = \left(\frac{r^n + \omega}{r^n + r}\right)!$ $\rightarrow \left\{ n = \frac{\omega - r^k}{r^k - r} \mid k \in \mathbb{N} \right\}$

$\frac{r^n + \omega}{r^n + r} \in \mathbb{N} \rightarrow n \in \frac{\omega - r^k}{r^k - r}$