روژانو صدرالملکی تکلیف ۲ : یازدهم ریاضی

۱-

$2x^2+y^2-4x+6y+k=0 \qquad k=?$

$(y+3)^2=y^2+6y+9$

$(y^2+6y+9)+(2x^2-4x+2)=0$

$+\ \boxed{11=k}$ ← جواب

$2x^2-4x+2=(\sqrt{2}x-\sqrt{2})^2$

$(\sqrt{2}x-\sqrt{2})^2=2x^2+2-4x$

۲-

$f(x)=\begin{cases} ax^2+4x & x\geq a \\ 2x+a+2 & x<a \end{cases} \qquad f(1)=?$

$a^2+4a=2a+a+2$

$a^2+a-2=0 \qquad (a+2)(a-1)=0$

$a=-2$ غ ق ق $\qquad a=1$ ✓

$f(x)\begin{cases} x^2+4x & x\geq 1 \\ 2x+3 & x<1 \end{cases} \qquad f(1)=1+4=\boxed{5}$ ← جواب

۳-

$f(x)\begin{cases} 2x^2-b & |x+1|\geq 2 \quad (x\geq 1,\ x\leq -3) \\ a+2x & -3<x<-1 \end{cases} \qquad a+b=?$

$x+1\geq 2$
$x\geq 1$

$x+1\leq -2$
$x\leq -3$

$2(-3)^2-b=a+2(-3)$

$18-b=a-6 \longrightarrow 18+6=a+b \Rightarrow \boxed{a+b=24}$ ← جواب

۴-

$y=\sqrt{4x-a}+\sqrt{b-3x} \qquad D_f=\{2\} \qquad \frac{b}{a}=?=\frac{6}{12}=\boxed{\frac{1}{2}}$ ← جواب

$4x\geq a \qquad b\geq 3x$

$x\geq \frac{a}{4} \qquad \frac{b}{3}\geq x \longrightarrow \frac{b}{3}\geq x\geq \frac{a}{4} \longrightarrow \frac{b}{3}\geq 2\geq \frac{a}{4} \qquad \frac{b}{3}=2 \quad b=6$

$\frac{a}{4}=2 \to a=12$

۵-

الف) $y=\sqrt{\frac{x+1}{|x-3|}\geq 0}+\sqrt{\frac{4-x}{x^2+2x+4}\geq 0}$

$-1 \qquad 3 \qquad 4$

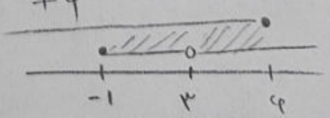
همواره مثبت باید باشد

	-1		3	
$-$		$+$		$+$

	4	
$+$		$-$

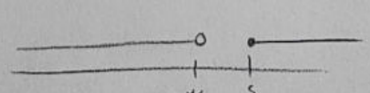
$\Delta=4-4\times 4\times 1=-12 \quad \Delta<0$

$a>0$ همواره مثبت



$D_f=[-1,3)\cup(3,4]$ ← جواب

ب) $y=\sqrt{[x]-4\geq 0}+\sqrt{2-[x]\geq 0}$



$[x]\geq 4 \qquad 2\geq [x]$

$[4,+\infty) \qquad (-\infty,3)$

$\boxed{D_f=\varnothing}$

۶-

الف) $y=\sqrt{\dfrac{x^2-x-6}{x^4-13x^2+36}}$ $\quad (x-3)(x+2)$

$(x^2-4)(x^2-9)$

$(x-2)(x+2)(x-3)(x+3)$

$\quad 2 \quad -2 \quad 3 \quad -3$

-3	-2	2	3

$+ \ \varnothing \ - \ \varnothing \ - \ \varnothing \ + \ \varnothing \ +$

$$\boxed{D_f=(-\infty,-3)\cup(2,3)\cup(3,+\infty)}$$

ب) $y=\sqrt{\dfrac{x^3-2x^2-5x+6}{x^3+2x^2-5x-6}}$

$1+(-2)+(-5)+6=0$

$1+(-5)=2(-2)$

$-4=-4$

$$\begin{array}{l|l} x^3-2x^2-5x+6 & x-1 \\ -(x^3-x^2) & \overline{x^2-x-6} \\ \hline -x^2-5x+6 & \\ -(-x^2+x) & \\ \hline -6x+6 & \\ -(-6x+6) & \\ \hline 0 & \end{array}$$

$(x-1)(x^2-x-6)=$

$(x-1)(x-3)(x+2)$

$$\begin{array}{l|l} x^3+2x^2-5x-6 & x+1 \\ -(x^3+x^2) & \overline{x^2+x-6} \\ \hline x^2-5x-6 & \\ -(x^2+x) & \\ \hline -6x-6 & \\ -(-6x-6) & \\ \hline 0 & \end{array}$$

$(x+1)(x^2+x-6)=$

$(x+1)(x+3)(x-2)$

$\Rightarrow \sqrt{\dfrac{(x-1)(x-3)(x+2)}{(x+1)(x+3)(x-2)}} \geq$

$\quad -1 \quad -3 \quad 2$

-3	-2	-1	1	2	3

$+ \ \varnothing \ - \ \bullet \ + \ \varnothing \ - \ \bullet \ + \ \varnothing \ - \ \bullet \ +$

$$\boxed{D_f=(-\infty,-3)\cup[-2,-1)\cup[1,2)\cup[3,+\infty)}$$

۷-

الف) $y=\sqrt{[-x]-2}\geq 0 \quad [-x]\geq 2$

$$\boxed{D_f=(-\infty,-2]}$$

ب) $y=\dfrac{5x^2+6}{[x]^2-2[x]-3}$

$(\underbrace{[x]}_{3}-3)(\underbrace{[x]}_{-1}+1)\neq 0$

$$\boxed{D_f=\mathbb{R}-\left([-1,0)\cup[3,4)\right)}$$

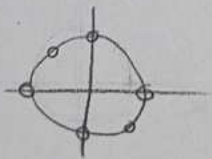
$[x]\neq 3 \quad x\neq[3,4)$

$[x]\neq -1 \quad x\neq[-1,0)$

۸-

الف) $y=\dfrac{\cot x+1}{\tan x+1}=\dfrac{\dfrac{\cos x}{\sin x}+1}{\dfrac{\sin x}{\cos x}+1}$

$\tan x\neq -1$



$$\boxed{D_f=\mathbb{R}-\left\{\frac{k\pi}{2},\ k\pi-\frac{\pi}{4}\right\}}$$

ب) $y=\sqrt{1-4\sin^2 x}\geq 0$ $\quad 1\geq 4\sin^2 x \quad \frac{1}{4}\geq \sin^2 x \quad \frac{1}{2}\geq \sin x\geq -\frac{1}{2}$

$$\boxed{D_f=\left[k\pi-\frac{\pi}{6},\ k\pi+\frac{\pi}{6}\right]}$$

۹-

الف) $y=\sqrt{1-\log_{\frac{1}{2}}(x-1)}\geq 0$

$\log_b a>c \xrightarrow{0<b<1} a<b^c$

$1\geq \log_{\frac{1}{2}}(x-1)$

① $x-1>0$ $\quad x>1$

$\frac{1}{2}\leq x-1$

② $\frac{3}{2}\leq x$

$$\boxed{D_f=\left[\frac{3}{2},+\infty\right)}$$

$$\boxed{D_f=(-\infty,0)\cup(3,+\infty)-\{-1,4\}}$$

$y=\dfrac{\sqrt{x^2-x}\geq 0}{1-\log_4(x^2-3x)}$ $\quad x(x-1)\geq 0$

$x(x-3)>0$

$1\neq \log_4(x^2-3x)$

$4\neq x^2-3x$

$0\neq x^2-3x-4 \quad 0\neq(x-4)(x+1) \quad x\neq\{-1,4\}$

۶ -

الف)

$$y=\sqrt{۲^{۴x-x^۲}-۸}\geqslant ۰$$

$$۲^{۴x-x^۲}-۸\geqslant ۰$$

$$۲^{۴x-x^۲}\geqslant ۸=۲^۳$$

$$۴x-x^۲\geqslant ۳$$

$$\Rightarrow x^۲-۴x+۳$$

$$\Rightarrow (x-۳)(x-۱)$$

$$x=۳ \quad x=۱$$

	۱		۳	
+		−		+

$$\underbrace{\quad}_{✓}$$

$$\boxed{D_f=[۱,۳]}$$

ب)

$$y=\left(\frac{۳x+۵}{۲x+۴}\right)!$$

$$\frac{۳x+۵}{۲x+۴}\in W$$

$$۳x+۵\in ۲xW+۴W$$

$$-۲xW+۳x\in ۴W-۵$$

$$x(-۲W+۳)\in ۴W-۵$$

$$\boxed{x\in \frac{۴W-۵}{-۲W+۳}}$$

$$\frac{۳x+۵}{۲x+۴}\in W$$

$$-\frac{۴W-۵}{۲W-۳}=\frac{۴W-۵}{-۲W+۳}$$

$$\boxed{D_f=\left\{x \,\middle|\, x=\frac{۴k-۵}{-۲k+۳},\ k\in W\right\}}$$