

$$\sqrt{x^2+y^2}-5x+4y+k=0 \rightarrow (\sqrt{x}-\sqrt{y})^2+(y+4)^2=0 \rightarrow \begin{cases} x=1 \\ y=-4 \end{cases}$$

تقاطع دایره و خط

$$(\sqrt{x}-\sqrt{y})^2 = x^2 + y^2 - 5x + 4y + k$$

$$(y+4)^2 = y^2 + 4y + 16$$

$$k = 2 + 9 = 11$$

$f(1) = ?$

$$(a > 0) \rightarrow f(x) = \begin{cases} x^2 + 5x & ; x \geq a \\ 2x + a + 2 & ; x < a \end{cases}$$

$x=a \rightarrow a^2 + 5a = 2a + 2$

$$a^2 + a - 2 = 0 \rightarrow \begin{cases} a = -2 \text{ (رد)} \\ a = 1 \text{ (قبول)} \end{cases}$$

$a=1 \rightarrow f(1) = f(a) = 1 + 5 = 6$

$a+b = ?$

$$|x+1| \geq 2$$

$$\begin{cases} 1) x+1 \geq 2 \rightarrow x \geq 1 \\ 2) x+1 \leq -2 \rightarrow x \leq -3 \end{cases}$$

$$x=-3 \rightarrow f(-3) = 2(-3)^2 - b = a + 2(-3)$$

$$18 - b = a - 6 \rightarrow a + b = 24$$

$$f(x) = \begin{cases} 2x^2 - b & ; |x+1| \geq 2 \\ a + 2x & ; -3 \leq x \leq -1 \end{cases}$$

$\frac{b}{a} = ?$

$$D_f = \{2\}$$

$$y = \sqrt{4x-a} + \sqrt{b-4x}$$

I: $4x-a \geq 0 \rightarrow x \geq \frac{a}{4}$

II: $b-4x \geq 0 \rightarrow x \leq \frac{b}{4}$

$$D_f = I \cap II = \{2\} \rightarrow \frac{a}{4} = \frac{b}{4} = 2 \rightarrow \begin{cases} b=8 \\ a=8 \end{cases} \rightarrow \frac{b}{a} = \frac{8}{8} = 1$$

الف) $y = \sqrt{\frac{x+1}{x-3}} + \sqrt{\frac{4-x}{x^2+2x+5}}$

I: $\frac{x+1}{x-3} \geq 0 \rightarrow x \in [-1, 3) \cup \{3\}$

II: $\frac{4-x}{x^2+2x+5} \geq 0 \rightarrow x \in (-\infty, 4]$

$D_f = I \cap II = [-1, 4] - \{3\}$

ب) $y = \sqrt{[x]-4} + \sqrt{4-[x]}$

I: $[x]-4 \geq 0 \rightarrow [x] \geq 4$

II: $4-[x] \geq 0 \rightarrow [x] \leq 4$

$D_f = I \cap II = \emptyset$

$\Delta = b^2 - 4ac$

$$\Delta = 4 - 4(1)(4) = -12$$

$\Delta < 0$: هیچ جوابی ندارد

$\Delta > 0$: دو جواب دارد

ا) $\sqrt{\frac{x^2-x-4}{x^2-13x^2+14x}} \geq 0 \rightarrow \sqrt{\frac{(x-4)(x+1)}{(x-1)(x-4)}} \geq 0 \rightarrow \begin{cases} x=4, 2 \\ t=4, 1 \rightarrow x=1, 4 \end{cases}$

$D_f: (-\infty, -1) \cup (1, 4) \cup (4, +\infty)$

ب) $\sqrt{\frac{x^3-2x^2-5x+4}{x^3+2x^2-5x-4}} \geq 0 \rightarrow \sqrt{\frac{(x-1)(x-4)(x+1)}{(x+1)(x+4)(x-2)}} \geq 0 \rightarrow \begin{cases} x=1, 4, -2 \\ x=-1, -4, 2 \end{cases}$

$D_f: (-\infty, -4) \cup (-1, 1) \cup [2, 4) \cup (4, +\infty)$

ا) $y = \sqrt{-x} - 2 \geq 0 \rightarrow [t] \geq 2 \rightarrow t \geq 2 \rightarrow -x \geq 2 \rightarrow x \leq -2$

$D_f: (-\infty, -2]$

ب) $y = \frac{2x^2+4}{[x]^2-2[x]-3} \neq 0 \rightarrow t^2-2t-3 \neq 0 \rightarrow (t-3)(t+1) \neq 0 \rightarrow \begin{cases} t \neq 3 \rightarrow [x] \neq 3 \\ t \neq -1 \rightarrow [x] \neq -1 \end{cases}$

$D_f: \mathbb{R} - ([3, 4) \cup [-1, 0))$

ا) $\frac{\cot x + 1}{\tan x + 1} = \frac{\cos x + \sin x}{\sin x} \neq 0$

$\begin{cases} \sin x \neq 0 \\ \cos x \neq 0 \\ \tan x \neq -1 \end{cases}$

$D_f: \mathbb{R} - \{k\pi, k\pi - \frac{\pi}{4}\}$

ب) $y = \sqrt{1 - \sin^2 x} \geq 0 \rightarrow \sin^2 x \leq 1$

$\frac{1}{x} \leq \sin x \leq \frac{1}{x}$

$D_f: [k\pi - \frac{\pi}{4}, k\pi + \frac{\pi}{4}]$

ا) $y = \sqrt{1 - \log_{\frac{1}{x}} x - 1} \geq 0$

$\begin{cases} x-1 > 0 \rightarrow x > 1: I \\ 1 - \log_{\frac{1}{x}} x \geq 0 \rightarrow 1 \geq \log_{\frac{1}{x}} x \\ \frac{1}{x} \leq x-1 \rightarrow \frac{1}{x} \leq x-1: II \end{cases}$

$I \cap II = [\frac{5}{4}, +\infty)$

ب) $y = \sqrt{\frac{x^2-x}{1 - \log_{\frac{1}{x}} x - 1}} \geq 0 \rightarrow \frac{x^2-x}{1 - \log_{\frac{1}{x}} x - 1} \geq 0$

$\begin{cases} x^2-x \geq 0 \rightarrow x(x-1) \geq 0 \rightarrow (-\infty, 0] \cup [1, +\infty): I \\ x \neq 1 \rightarrow x \neq 1 \\ x^2-x > 0 \rightarrow x > 1 \end{cases}$

$D_f: I \cap II \cap III = (-\infty, 0) \cup (1, +\infty) - \{1\}$

ا) $y = \sqrt{y^2 x - x^2 - 1} \geq 0$

$y^2 x - x^2 \geq 1 \rightarrow x - x^2 \geq \frac{1}{y^2}$

$x^2 - x + \frac{1}{y^2} \leq 0 \rightarrow (x-1)(x-\frac{1}{y^2}) \leq 0$

$D_f: [1, \frac{1}{y^2}]$

ب) $y = \left(\frac{rx+d}{rx+f} \right)!$

$\frac{rx+d}{rx+f} \in \mathbb{N} \rightarrow x \in \mathbb{N} - \frac{f\omega - d}{r\omega - r}$

$D_f: \left\{ x \mid x = \frac{r\omega + d}{r\omega - f}, \omega \in \mathbb{N} \right\}$