

$$x^2 + y^2 - 4x + 4y + k = 0 \xrightarrow{\text{تکمیل مربع}} 2(x-1)^2 + (y+2)^2 = 0$$

$$\Rightarrow x^2 + 2 - 4x + y^2 + 4y + 4 = k \Rightarrow x^2 + y^2 - 4x + 4y + 6 = k \quad \text{II}$$

لج-تک-نقطة (۱، -۳)

$$f(x) = \begin{cases} x^2 + 4x & ; x \geq a \\ x^2 + a + 2 & ; x < a \end{cases} \xrightarrow{x=a} a^2 + 4a = a^2 + a + 2 \Rightarrow a^2 + a + 2 = 0$$

$$\rightarrow (a+2)(a-1) = 0 \begin{cases} a=1 \\ a=-2 \end{cases} \xrightarrow{a \geq 0} \boxed{a=1} \Rightarrow f(x) = \begin{cases} x^2 + 4x & ; x \geq 1 \\ x^2 + 3 & ; x < 1 \end{cases}$$

$$f(1) = 1^2 + 4 = 5 \quad f(1) = \boxed{5}$$

$$f(x) = \begin{cases} x^2 - b & ; |x+1| \geq 2 \\ a + 2x & ; -3 \leq |x+1| < 2 \end{cases} \begin{cases} x=3 \\ x=-5 \end{cases} \Rightarrow \frac{3(-3)^2 - b}{1} = a + 2(-3) \Rightarrow \boxed{a+b = 24}$$

$$y = \sqrt{4x-a} + \sqrt{b-3x}$$

$$\begin{aligned} 4x-a &\xrightarrow{x=2} 12-a=0 \rightarrow \boxed{a=12} \\ b-3x &\xrightarrow{x=2} b-6=0 \rightarrow \boxed{b=6} \end{aligned}$$

$$\boxed{\frac{b}{a} = \frac{6}{12} = \frac{1}{2}}$$

$$\text{الف) } y = \sqrt{\frac{x+1}{|x-1|}} + \sqrt{\frac{4-x}{x^2+x+2}} \quad \begin{cases} \frac{x+1}{|x-1|} \geq 0 \rightarrow \frac{-1}{-1} \leq \frac{x}{-1} \leq \frac{1}{-1} \rightarrow x \leq -1 \\ x^2+x+2 > 0 \rightarrow \frac{-1 \pm \sqrt{1-8}}{2} < x < \frac{-1 \pm \sqrt{1-8}}{2} \end{cases} \Rightarrow D_f = [1, 4] - \{-1\}$$

$$\text{ب) } y = \sqrt{[x]-2} + \sqrt{2-[x]} \quad \begin{cases} [x]-2 \geq 0 \rightarrow [x] \geq 2 \rightarrow x \geq 2 \\ 2-[x] \geq 0 \rightarrow [x] \leq 2 \rightarrow x \leq 2 \end{cases} \rightarrow \emptyset \Rightarrow \boxed{D_f = \emptyset}$$

$$a) y = \sqrt{\frac{x^2 - x - 4}{x^2 - 15x^2 + 4}} \Rightarrow \frac{(x-2)(x+2)}{(x-2)(x+2)(x+2)(x+2)} \geq 0 \quad \begin{array}{c} -2 \quad -2 \quad 2 \quad 2 \\ + \quad - \quad - \quad + \quad + \end{array}$$

$$\Rightarrow D_f = (-\infty, -2) \cup (2, +\infty) - \{2\}$$

$$b) y = \sqrt{\frac{x^2 - 2x^2 - 2x + 4}{x^2 + 2x^2 - 2x - 4}} \Rightarrow \frac{(x-1)(x-2)(x+2)}{(x+1)(x+2)(x-2)} \geq 0 \quad \begin{array}{c} -2 \quad -2 \quad -1 \quad 1 \quad 2 \quad 2 \\ + \quad - \quad - \quad + \quad - \quad + \end{array}$$

$$D_f = (-\infty, -2) \cup [-2, -1) \cup [1, 2) \cup [2, +\infty)$$

$$a) y = \sqrt{[-x] - 2} \rightarrow [-x] - 2 \geq 0 \rightarrow [-x] \geq 2 \rightarrow -x \geq 2 \rightarrow x \leq -2 \quad \begin{array}{c} -2 \\ D_f = (-\infty, -2] \end{array}$$

$$b) y = \frac{\sin^2 x + 4}{[x]^2 - 2[x] - 2} \rightarrow [x]^2 - 2[x] - 2 \neq 0 \rightarrow ([x] - 2)([x] + 1) \neq 0$$

$$\begin{array}{l} [x] - 2 \neq 0 \rightarrow [x] \neq 2 \rightarrow x \notin [2, 3) \\ [x] + 1 \neq 0 \rightarrow [x] \neq -1 \rightarrow x \notin [-1, 0) \end{array}$$

$$D_f = \mathbb{R} - ([2, 3) \cup [-1, 0))$$

$$a) y = \frac{\cot x + 1}{\tan x + 1} \Rightarrow \begin{array}{l} \sin x \neq 0 \\ \cos x \neq 0 \\ \tan x + 1 \neq 0 \rightarrow \tan x \neq -1 \end{array} \quad \begin{array}{c} \text{Unit Circle} \end{array} \Rightarrow D_f = \mathbb{R} - \left\{ \frac{k\pi}{2}, k\pi - \frac{\pi}{4} \right\}$$

$$b) y = \sqrt{1 - \sin^2 x} \Rightarrow 1 - \sin^2 x \geq 0 \rightarrow \sin^2 x \leq 1 \rightarrow -1 \leq \sin x \leq 1$$

$$\begin{array}{c} \text{Unit Circle} \end{array} \Rightarrow D_f = \left[k\pi - \frac{\pi}{2}, k\pi + \frac{\pi}{2} \right]$$

$$a) y = \sqrt{1 - \log \frac{x-1}{x}} \Rightarrow \begin{array}{l} x-1 > 0 \rightarrow x > 1 \\ 1 - \log \frac{x-1}{x} \geq 0 \Rightarrow \log \frac{x-1}{x} \leq 1 \rightarrow x-1 \leq x \rightarrow x \geq \frac{x}{x-1} \end{array} \Rightarrow D_f = \left[\frac{x}{x-1}, +\infty \right)$$

$$b) y = \frac{\sqrt{x^2 - x}}{1 - \log \frac{x^2 - x}{x}} \Rightarrow \begin{array}{l} x^2 - x \geq 0 \rightarrow x(x-1) \geq 0 \\ 1 - \log \frac{x^2 - x}{x} \neq 0 \rightarrow \log \frac{x^2 - x}{x} \neq 1 \rightarrow x^2 - x \neq x \\ x^2 - x - x \neq 0 \rightarrow (x-2)(x+1) \neq 0 \rightarrow x \neq \{2, -1\} \\ x^2 - x > 0 \rightarrow x(x-1) > 0 \end{array} \Rightarrow D_f = (-\infty, 0) \cup (1, +\infty) - \{2, -1\}$$

$$a) y = \sqrt{x^{2n-2} - 1} \rightarrow x^{2n-2} - 1 \geq 0 \rightarrow x^{2n-2} \geq 1 \rightarrow (x-1)(x+1) \geq 0 \quad \begin{array}{c} 1 \quad -1 \\ - \quad + \quad - \quad + \end{array} \Rightarrow D_f = [1, \infty)$$

$$\hookrightarrow \left(\frac{x_n + \Delta}{x_n + \Gamma} \right)! \rightarrow \frac{x_n + \Delta}{x_n + \Gamma} \in \mathbb{W} \rightarrow x \in \frac{-\Gamma \mathbb{W} + \Delta}{\Gamma \mathbb{W} - \Gamma}$$

$$\Rightarrow D_f = \left\{ x \mid x = \frac{\Delta - \Gamma k}{\Gamma k - \Gamma}, k \in \mathbb{W} \right\}$$