۱-
$$2x^2+y^2-4x+4y+k=0$$
$$2(x^2)(2)=2(\sqrt{2}x)(\sqrt{2}x)$$
$$(\sqrt{2}x-\sqrt{2})^2+(y+2)^2=0 \Rightarrow 2x^2-4x+2+\underbrace{y^2+4y+9}_{k}=0 \Rightarrow k=9+2=(11)$$

۲-
$$f(x)=\begin{cases} x^2+4x & ; x\geq -1 \\ 2x+2+2 & ; x\leq -1 \end{cases}$$
$$a^2+4a=2a+a+2$$
$$a^2+a-2=0 \Rightarrow (a+2)(a-1)=0 \Rightarrow a=1 \quad (a>0)$$
$$f(1)=2+3=(5)$$

۳-
$$f(x)=\begin{cases} 2x^2-b & ; |x+1|\geq 2 \Rightarrow -2\geq x+1\geq 2 \Rightarrow -3\geq x\geq 1 \\ a+2x & ; -3\leq x\leq -1 \end{cases}$$
$$x=-3 \Rightarrow 18-b=a-6 \Rightarrow a+b=(24)$$

۴-
$$y=\sqrt{4x-a}+\sqrt{b-2x}$$
$$4x-a\geq 0 \rightarrow x\geq \frac{a}{4}$$
$$b-2x\geq 0 \rightarrow x\leq \frac{b}{2}$$
$$\frac{a}{4}=\frac{b}{2}=2 \Rightarrow \frac{b}{a}=\frac{2}{4}=(\frac{1}{2})$$

۵-
الف) $y=\sqrt{\frac{x+1}{|x-2|}}+\sqrt{\frac{4-x}{x^2+2x+2}}$

$\frac{x+1}{|x-2|}\geq 0$: $\frac{-1\ (*)}{-\ \phi\ +\ \phi\ +}$ $\rightarrow [-1,2)\cup(2,+\infty)$

$\frac{4-x}{x^2+2x+2}\geq 0$: $\frac{4}{+\ \phi\ -}$ $\rightarrow (-\infty,4]$

$\xrightarrow{\cap} [-1,2)\cup(2,4]$

ب) $y=\sqrt{[x]-4}+\sqrt{2-[x]}$

$[x]-4\geq 0 \Rightarrow [x]\geq 4 \rightarrow [4,+\infty)$

$2-[x]\geq 0 \Rightarrow 2\geq [x] \rightarrow (-\infty,3)$

$\xrightarrow{\cap} \varnothing$

۶-
الف) $\sqrt{\frac{x^2-x-6}{x^4-13x^2+36}}$

$x^2-x-6 \rightarrow (x-3)(x+2)$

$x^4-13x^2+36 \rightarrow (x^2-9)(x^2-4)=(x-3)(x+3)(x-2)(x+2)$

$\frac{-3\ (*)\quad -2\ (*)\quad 2\ (*)\quad 3\ (*)}{+\ \phi\ -\ \phi\ -\ \phi\ +\ \phi\ +}$

$(-\infty,-3)\cup(-2,2)\cup(3,+\infty)$

ب) $y = \sqrt{\frac{x^3-2x^2-5x+4}{x^3+2x^2-5x-4}}$ $(x-1)(x-4)(x+2)$ / $(x+1)(x+4)(x-2)$

$x^3-2x^2-5x+4 \mid x-1$; $x^3+2x^2-5x-4 \mid x+1$

x^2-x-4 ; x^2+x-4

$-x^3-5x+4$; $-x^3-5x-4$

$+x^2+x$; $+2x^2+x$

$-4x+4$; $-4x-4$

$+4x-4$; $+4x+4$

0 ; 0

$-3, -2, -1, 1, 2, 3$

$+ \phi - \phi + \phi - \phi +$

$\rightarrow (-\infty, -3) \cup [-2, -1) \cup [1, 2) \cup [3, +\infty)$

-۷

الف) $y = \sqrt{[-x]-2}$ $\quad [-x]-2 \geq 0$

$[-x] \geq 2 \Rightarrow (-\infty, -2]$

ب) $\frac{5x^2+4}{[x]^2-2[x]-3}$ $\quad [x]^2-2[x]-3 \neq 0$

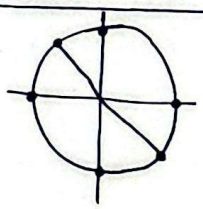
$(x-3)(x+1) \neq 0$ $\quad \Rightarrow R-([-1, 0) \cup [3, 4))$

-۸

الف) $y = \frac{\cot x + 1}{\tan x + 1}$ $\quad \sin x \neq 0$

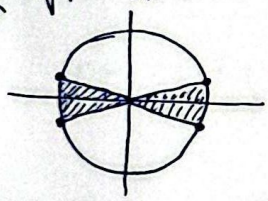
$\cos x \neq 0$

$\tan x + 1 \neq 0 \Rightarrow \tan x \neq -1$



$R - \left\{ \frac{k\pi}{2}, k\pi - \frac{\pi}{4} \right\}$

ب) $y = \sqrt{1-4\sin^2 x}$ $\quad 1-4\sin^2 x \geq 0$ $\quad 1 \geq 4\sin^2 x$ $\quad \sin^2 x \leq \frac{1}{4} \Rightarrow -\frac{1}{2} \leq \sin x \leq \frac{1}{2}$



$\left[k\pi - \frac{\pi}{6}, k\pi + \frac{\pi}{6} \right]$

-۹

الف) $y = \sqrt{1 - \log_{\frac{1}{2}}^{x-1}}$ $\quad x-1 > 0 \Rightarrow x > 1$

$1 - \log_{\frac{1}{2}}^{x-1} \geq 0 \Rightarrow \log_{\frac{1}{2}}^{x-1} \leq 1$ $\quad x-1 \geq \frac{1}{2} \Rightarrow x \geq \frac{3}{2}$

$[\frac{3}{2}, +\infty)$

ب) $y = \frac{\sqrt{x^2-x}}{1 - \log_4^{x^2-3x}}$

$x^2-x \geq 0$ $\quad x(x-1) \geq 0$ $\quad + \phi - \phi +$ $\quad (-\infty, 0] \cup [1, +\infty)$

$x^2-3x > 0$ $\quad x(x-3) > 0$ $\quad + \phi - \phi +$ $\quad (-\infty, 0) \cup (3, +\infty)$

$1 - \log_4^{x^2-3x} \neq 0$ $\quad \log_4^{x^2-3x} \neq 1$ $\quad x^2-3x \neq 4$

$x^2-3x-4 \neq 0$ $\quad (x-4)(x+1) \neq 0$

$x \neq 4, -1$

$((-\infty, 0) \cup (3, +\infty)) - \{-1, 4\}$

۱۵ -

الف) $y = \sqrt{۳^{۴n-n^۲} - ۸}$ $\quad ۳^{۴n-n^۲} - ۸ \geq ۰$ $\quad ۳^{۴n-n^۲} \geq ۳^۳$

$۴n - n^۲ \geq ۳$ $\quad n^۲ - ۴n + ۳ \leq ۰$ $\quad (n-۳)(n-۱) \leq ۰$ $\quad \frac{۱ \quad ۳}{+ \; - \; +}$ $\quad [۱, ۳]$

ب) $y = \left(\frac{۳n+۵}{۲n+۴}\right)!$ $\quad \frac{۳n+۵}{۲n+۴} \in W \Rightarrow ۳n+۵ \in ۲nW + ۴W$

$۳n - ۲nW \in ۴W - ۵$

$n(۳-۲W) \in ۴W - ۵ \Rightarrow n \in \frac{۴W-۵}{۳-۲W}$ $\quad \left\{n \mid n = \frac{۴k-۵}{۳-۲k}, k \in W\right\}$