Subject:
Year. Month. Date. ( )

تکلیف ۲ — دریا رمانی درودشتی

(۱)

$2x^2 + y^2 - 4x + 6y + k = 0 \longrightarrow 2x^2 - 4x + k_1 = -6y - y^2$

$y^2 + 6y + k = -2x^2 + 4x$

$(\sqrt{2}x - \sqrt{2})^2 \longrightarrow k = 2$ ، $2 + 9 = 11$

می‌خواهیم که کامل مربع بشود پس باید عدد ۹ باشد

$\longrightarrow y^2 + 6y + 9 = -2x^2 + 4x \longrightarrow (y+3)^2 = -2x^2 + 4x$

$k = 9$ جواب

(۲)

$f(x) = \begin{cases} x^2 + 4x & x \geq a \longrightarrow a^2 + 4a \\ 2x + a + 2 & x < a \longrightarrow 2a + a + 2 = 3a + 2 \end{cases}$

$a^2 + 4a = 3a + 2$

$a^2 + a - 2 = (a+2)(a-1)$

$a = -2 , a = 1$ ، $a > 0$ ⟹ $a = 1$

$f(1) = f(a) = 3a + 2 = 3 \times 1 + 2 = 5$ جواب

(۳)

$f(x) = \begin{cases} 2x^2 - b & |x+1| \geq 2 \\ a + 2x & -3 < x < 1 \end{cases}$

$\longrightarrow x + 1 \geq 2 \longrightarrow x \geq 1$

$\longrightarrow x + 1 \leq -2 \longrightarrow x \leq -3$

$f(-3) = 2x^2 - b = 2 \times (-3)^2 - b = 18 - b$

$f(-3) = a + 2x = a + 2(-3) = a - 6$

$18 - b = a - 6 \longrightarrow a + b = 24$ جواب

(۴)

$y = \sqrt{6x - a} + \sqrt{b - 3x}$ ، $D_f = \{2\}$

$6x - a \geq 0 \longrightarrow 6x \geq a \longrightarrow x \geq \frac{a}{6}$

$b - 3x \geq 0 \longrightarrow 3x \leq b \longrightarrow x \leq \frac{b}{3}$

$\frac{b}{3} = \frac{a}{6} = 2 \longrightarrow 6b = 3a \longrightarrow \frac{b}{a} = \frac{1}{2}$ جواب

(۵)

الف) $y = \sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{4-x}{x^2+2x+4}} \longrightarrow (x+1)^2 + 3 \Longrightarrow$ همواره مثبت

$\frac{x+1}{|x-3|} \geq 0$ (همواره نامنفی) $\quad -1 \quad 3 \quad - \; + \; + \longrightarrow [-1\ 3)\ (3\ +\infty)$

$\frac{4-x}{(x+1)^2+3} \geq 0 \quad 4 \quad + \; - \longrightarrow (-\infty\ 4]$

$D_f = [-1\ 3) \cup (3\ 4]$ جواب

ب) $\sqrt{[x] - 4} + \sqrt{2 - [x]}$

$[x] - 4 \geq 0 \longrightarrow [x] \geq 4 \longrightarrow [4\ +\infty)$

$2 - [x] \geq 0 \longrightarrow [x] \leq 2 \longrightarrow (-\infty\ 3)$

$(-\infty\ 3) \cap [4\ +\infty) = \varnothing$

$D_f = \varnothing$ جواب

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الف) $y = \sqrt{\dfrac{x^2-x-6}{x^4-13x^2+36}}$ → $(x-3)(x+2)$ $\left[\begin{matrix} x=3 \\ x=-2 \end{matrix}\right.$

$x^4-13x^2+36 \to t^2-13t+36 \to (t-9)(t-4)$ → $t=9 \to x^2=9 \to x=\pm 3$; $t=4 \to x^2=4 \to x=\pm 2$

$-3 \quad -2 \quad 2 \quad 3$: $+ \; - \; + \; - \; +$ → $(-\infty \;\; -3) \cup (2 \;\; 3) \cup (3 \;\; +\infty) = D_f$

ب) $y = \sqrt{\dfrac{x^3-2x^2-5x+6}{x^3+2x^2-5x-6}}$ → ۰ جمع ضرایب → $(x^3-2x^2-5x+6) \div (x-1) = x^2-x-6$

→ جمع ضرایب فرد = جمع ضرایب زوج → $(x^3+2x^2-5x-6) \div (x+1) = x^2+x-6$

$\dfrac{(x-1)(x-3)(x+2)}{(x+1)(x+3)(x-2)} \geq 0$; $-3 \quad -2 \quad -1 \quad 1 \quad 2 \quad 3$: $+ \; - \; + \; - \; + \; - \; +$ → $D_f = (-\infty \;\; -3) \cup [-2 \;\; -1) \cup [1 \;\; 2) \cup [3 \;\; +\infty)$

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الف) $y = \sqrt{[-x]-2}$ → $[-x]-2 \geq 0 \to [-x] \geq 2 \to x \leq -2$

$[x] \leq -2$; $D_f = (-\infty \;\; -2]$

ب) $y = \dfrac{5x^2+6}{[x]^2-2[x]-3}$

$[x]^2-2[x]-3 \neq 0 \to ([x]-3)([x]+1)$

$[x] \neq 3$ → $-1 \quad 0 \quad 3 \quad 4$; $R - \{[-1 \;\; 0) \cup [3 \;\; 4)\} = D_f$

$[x] \neq -1$

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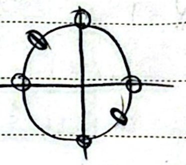
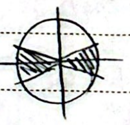
الف) $\dfrac{\cot x+1}{\tan x+1} \to \dfrac{\frac{\cos x}{\sin x}+1}{\frac{\sin x}{\cos x}-1}$; $\tan x \neq -1$; $\sin x \neq 0$; $\cos x \neq 0$



$D_f = \mathbb{R} - \{2k\pi,\; 2k\pi+\frac{\pi}{2},\; 2k\pi+\frac{3\pi}{4},\; 2k\pi+\pi,\; 2k\pi+\frac{3\pi}{2},\; 2k\pi+\frac{7\pi}{4}\}$

ب) $\sqrt{1-4\sin^2 x} \to 1-4\sin^2 x \geq 0 \to 4\sin^2 x \leq 1 \to \sin^2 x \leq \frac{1}{4} \to -\frac{1}{2} \leq \sin x \leq \frac{1}{2}$

$D_f = [2k\pi-\frac{\pi}{6},\; 2k\pi+\frac{\pi}{6}] \cup [2k\pi+\frac{5\pi}{6},\; 2k\pi+\frac{7\pi}{6}]$



۹

الف) $y = \sqrt{1-\log_{\frac{1}{2}}^{x-1}}$; $1-\log_{\frac{1}{2}}^{x-1} \geq 0 \to \log_{\frac{1}{2}}^{x-1} \leq 1 \to x-1 \geq \frac{1}{2} \to x \geq \frac{3}{2}$

(1) $x-1>0 \to x>1$; $D_f = [\frac{3}{2},\; +\infty)$

ب) $y = \dfrac{\sqrt{x^2-x}}{1-\log_4^{x^2-3x}}$

1) $x^2-x \geq 0 \to x(x-1)$; $+ \; - \; +$ → $(-\infty \;\; 0] \cup [1 \;\; +\infty)$

2) $x^2-3x > 0 \to x(x-3)$; $+ \; - \; +$ → $(-\infty \;\; 0) \cup (3 \;\; +\infty)$

3) $1-\log_4^{x^2-3x} \neq 0 \to \log_4^{x^2-3x} \neq 1$

$x^2-3x \neq 4 \to x^2-3x-4 \neq 0$

$(x-4)(x+1) \to x \neq 4$; $x \neq -1$

$D_f = (-\infty \;\; -1) \cup (-1 \;\; 0) \cup (3 \;\; 4) \cup (4 \;\; +\infty)$

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الف) $y = \sqrt{2^{4x-x^2}-8} \to 2^{4x-x^2} - 2^3 \geq 0$

$2^{4x-x^2} \geq 2^3 \to 4x-x^2 \geq 3 \to x^2-4x+3 \leq 0$

$(x-3)(x-1) \leq 0$; $1 \quad 3$: $+ \; - \; +$

$D_f = [1 \;\; 3]$

ب) $\left(\dfrac{3x+5}{2x+4}\right)! \to \dfrac{3x+5}{2x+4} \in W$

$3x+5 = 2xw+4w$

$3x-2xw = 4w-5$

$x(3-2w) = 4w-5 \to x = \dfrac{4w-5}{3-2w}$

$D_f = \{x \mid x = \dfrac{4k-5}{3-2k},\; k \in W\}$

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