$3x^2 - 7x + 4$ | + + − 0 + | $[\frac{3}{2}, +\infty)$

تکلیف شماره ۲ — دینا جمشیدی — یازدهم دختر — ۱۹

۱) تابع ناتی

$2x^2 + y^2 - 4x + 6y + k = 0$ →

$2x^2 - 4x \to 2(x^2 - 2x + 1) = 2x^2 - 4x + 2$

$y^2 + 6y \to y^2 + 6y + 9$

$k = 9 + 2 = 11$

$2(x-1)^2 + (y+3)^2 = 0 = k - 11 \to k = 11$

۱,۵

۲)

$f(x) = \begin{cases} x^2 + 4x ; & x \geq a \\ 2x + a + 2 ; & x < a \end{cases}$

$x = a \to a^2 + 4a = 3a + 2$

$a^2 + a - 2 = 0 \quad (a-1)(a+2) = 0$

$f(1) = ?$

$\left.\begin{array}{l} a = 1 \\ a = -2 \end{array}\right\} a > 0 \to a = 1$

$f(1) = 1 + 4 = 5$

✓ ۵

s.a.m

۳) $f(x)=\begin{cases} 3x^2-b & ; \ |x+1|\geq 2 \\ a+2x & ; \ -3\leq x<-1 \end{cases}$

$|x+1|\geq 2 \rightarrow \begin{cases} x+1\geq 2 & x\geq 1 \\ x+1\leq -2 & x\leq -3 \end{cases}$

$x=-3 \rightarrow 3x^2-b=a+2x$

$18-b=a-6 \rightarrow a+b=24$

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۴) $y=\sqrt{6x-a}+\sqrt{b-3x}$ $\quad D_f=\{2\}$ $\quad \frac{b}{a}=? \quad \frac{6}{12}=\frac{1}{2}$

$6x-a\geq 0 \quad 6x\geq a \quad x\geq \frac{a}{6}$ $\quad \frac{a}{6}=2 \quad a=12$

$b-3x\geq 0 \quad 3x\leq b \quad x\leq \frac{b}{3}$ $\quad \frac{b}{3}=2 \quad b=6$

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۵) الف) $y=\sqrt{\frac{x+1}{|x-3|}}+\sqrt{\frac{6-x}{x^2+2x+4}}$ $\quad D_f=?=[-1,6]-\{3\}$

$\frac{x+1}{|x-3|}\geq 0$ (ریشه‌ها: -1، 3)

	-1		3	
$-$	0	$+$	ت.ن	$+$

$\frac{6-x}{x^2+2x+4}\geq 0$ $\quad \Delta<0,\ a>0 \rightarrow$ همواره $+$

	6	
$+$	0	$-$

محور: -1، 3، 6

ب) $y=\sqrt{[x]-4}+\sqrt{2-[x]}$ $\quad D_f=?=\varnothing$

$[x]-4\geq 0 \quad [x]\geq 4 \quad x\geq 4$ $\qquad 2-[x]\geq 0 \quad [x]\leq 2 \quad x<3 \xrightarrow{\cap} \varnothing$

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۶) الف) $\sqrt{\frac{x^2-x-6}{x^4-13x^2+36}} \Rightarrow \frac{(x+2)(x-3)}{(x^2-4)(x^2-9)}=\frac{(x+2)(x-3)}{(x-2)(x+2)(x+3)(x-3)}\geq 0$

	-3		-2		2		3	
$+$	ت.ن	$-$	ت.ن	$-$	ت.ن	$+$	ت.ن	$+$

$D_f=(-\infty,-3)\cup(2,3)\cup(3,+\infty)$

s.a.m

ب) $y=\sqrt{\frac{x^3-2x^2-5x+6}{x^3+2x^2-5x-6}}$

جمع ضرایب صفر $\rightarrow$

$x^3-2x^2-5x+6 \div (x-1) = x^2-x-6 = (x+2)(x-3)$

جمع ضرایب مرتبه فرد = " " زوج

$x^3+2x^2-5x-6 \div (x+1) = x^2+x-6 = (x-2)(x+3)$

$\frac{(x-1)(x+2)(x-3)}{(x+1)(x-2)(x+3)}\geq 0$

	-3		-2		-1		1		2		3	
$+$	ت.ن	$-$	0	$+$	ت.ن	$-$	0	$+$	ت.ن	$-$	0	$+$

$D_f=(-\infty,-3)\cup(-2,-1)\cup(1,2)\cup(3,+\infty)$

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۷) الف) $y=\sqrt{[-x]-2}$ $\quad D_f=(-\infty,-2]$

$[-x]-2\geq 0 \quad [-x]\geq 2 \quad -x\geq 2 \quad x\leq -2$

ب) $y=\frac{5x^2+6}{[x]^2-2[x]-3}$ $\quad [x]^2-2[x]-3\neq 0$

$([x]+1)([x]-3)\neq 0 \begin{cases} [x]\neq -1 & x\neq (-2,-1] \\ [x]\neq 3 & x\neq (2,3] \end{cases}$

$x\neq[-1,0)$ $\qquad x\neq[3,4)$

$D_f=\mathbb{R}-(-2,-1]\cup(2,3]$

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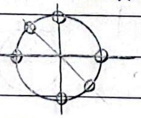
۸) الف) $y=\frac{\cot x+1}{\tan x+1}$

$\frac{\cos x}{\sin x}\rightarrow \sin x\neq 0$

$\frac{\sin x}{\cos x}\rightarrow \cos x\neq 0$ $\qquad D_f=\mathbb{R}-\{\frac{k\pi}{2},\ k\pi-\frac{\pi}{4}\}$

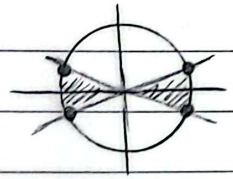


$\tan x+1\neq 0 \qquad \tan x\neq -1$

s.a.m

ب) $y = \sqrt{1 - 4\sin^2 x}$

$1 - 4\sin^2 x \geq 0 \quad 4\sin^2 x \leq 1 \quad \sin^2 x \leq \frac{1}{4} \quad -\frac{1}{2} \leq \sin x \leq \frac{1}{2}$



$D_f = [k\pi - \frac{\pi}{6}, k\pi + \frac{\pi}{6}]$

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۹)

الف) $y = \sqrt{1 - \log_{\frac{1}{2}}^{x-1}} \quad D_f = [\frac{3}{2}, +\infty)$

$1 - \log_{\frac{1}{2}}^{x-1} \geq 0 \quad \log_{\frac{1}{2}}^{x-1} \leq 1 \quad x - 1 \geq \frac{1}{2} \quad x \geq \frac{3}{2}$

ب) $\frac{\sqrt{x^2 - x}}{1 - \log_4^{x^2 - 3x}}$

$x^2 - x \geq 0 \quad x(x-1) \geq 0$ (② ①)

	0	1	
	+	−	+

$1 - \log_4^{x^2-3x} \neq 0 \quad \log_4^{x^2-3x} \neq 1 \quad x^2 - 3x \neq 4$

$x^2 - 3x - 4 \neq 0 \quad (x+1)(x-4) \neq 0$

$x \neq -1, 4 \longleftarrow$

$D_f = (-\infty, 0] \cup [1, +\infty) - \{-1, 4\}$

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۱۰)

الف) $y = \sqrt{2^{4x - x^2} - 7}$

$2^{4x - x^2} - 7 \geq 0$

$2^{4x - x^2} \geq 7$

$4x - x^2 \geq 3 \quad x^2 - 4x + 3 \leq 0$

$(x-1)(x-3) \leq 0$ (① ③)

	1	3	
	+	−	+

$D_f = [1, 3]$

ب) $y = \left(\frac{3x + 5}{2x + 4}\right)!$

$\frac{3x+5}{2x+4} \in \omega$

$3x + 5 \in 2\omega x + 4\omega + 3$

$\omega + 4 \in 2\omega x + 4\omega$

$\omega - 4 \in x(2\omega - 3)$

$x \in \frac{\omega - 4}{2\omega - 3}$

$D_f = \{x \mid x = \frac{4k - 5}{3k - 2}, k \in \omega\}$

s.a.m