شهرزاد شوشتری ۲

۱)

$2x^2+y^2-4x+6y+k=0 \to (\sqrt{2}x-\sqrt{2})^2+(y+3)^2 \to 2x^2+2+4x+y^2+9+6y$

$k=9+2=11$

۲)

$a^2+4a=2a+a+2 \to a^2+4a=3a+2 \to a^2-a-2=0 \to (a-2)(a+1)$

$a=2$, $a=-1$ غ ق ق

$1<2 \to 2+2+2=6$

۳)

$-3 \to 2\times 9-b \to 18-b \quad 18-b=a-6$

$3 \to a-6 \to a-6 \quad 24=a+b \to a+b=24$

۴)

$6x-a \geq 0 \to 6x \geq a \to x \geq \frac{a}{6} \to D_f=[\frac{a}{6},+\infty) \quad \frac{a}{6}=2 \to a=12$

$b-3x \geq 0 \to 3x \leq b \to x \leq \frac{b}{3} \to D_f=(-\infty,\frac{b}{3}) \quad \frac{b}{3}=2 \to b=6$

$\frac{b}{a}=\frac{6}{12}=\frac{1}{2}$

۵) الف

$y=\sqrt{\frac{x+1}{|x-3|}}+\sqrt{\frac{6-x}{x^2+2x+4}} \to \frac{-1}{-\ \phi\ +} \to D_f[-1,+\infty]-\{3\}$

همواره مثبت

$4-16<0$ (زیر رادیکال)

$\frac{6}{+\ \phi\ -} \to D_f=(-\infty,6]$

$\xrightarrow{\cap} [-1,6]-\{3\}$

ب) $y=\sqrt{[x]-4}+\sqrt{2-[x]}$ $\quad [x]-4 \geq 0 \to [x] \geq 4 \xrightarrow{D_f} [4,+\infty)$

$2-[x] \geq 0 \to [x] \leq 2 \to D_f(-\infty,3) \xrightarrow{\cap} \emptyset$

۶) الف $=\sqrt{\frac{x^2-x-6}{x^4-13x^2+36}}$ $\quad \sqrt{\frac{(x-3)(x+2)}{t^2-13t+36}}$ (3, −2)

$t^2-13t+36 \to (t-9)(t-4)$

$x^2=9 \to x=\pm 3 \quad x^2=4 \to x=\pm 2$

$-3 \quad -2^* \quad 2 \quad 3^*$

$+\ \phi\ -\ \phi\ -\ \phi\ +\ \phi\ +$

$(-\infty,-3)\cup(2,3)\cup(3,+\infty)$

شهرزاد شوشتری ۲

$$y=\sqrt{\frac{x^3-2x^2-5x+6}{x^3+2x^2-5x-6}}$$

$$\begin{array}{l} x^3-2x^2-5x+6 \;\big|\; x-1 \\ -x^3+x^2 \qquad\qquad\quad x^2-x-6 \\ \hline -x^2-5x+6 \\ +x^2-x \\ \hline -6x+6 \\ +6x-6 \\ \hline 0 \end{array} \qquad \begin{array}{l} x^3+2x^2-5x-6 \;\big|\; x+1 \\ -x^3-x^2 \qquad\qquad\quad x^2+x-6 \\ \hline x^2-5x-6 \\ -x^2-x \\ \hline -6x-6 \\ +6x+6 \\ \hline \end{array}$$

$$y=\sqrt{\frac{(x-1)(x^2-x-6)}{(x+1)(x^2+x-6)}} \rightarrow \sqrt{\frac{(x-1)(x-3)(x+2)}{(x+1)(x+3)(x-2)}}$$

Roots: $1,\ 3,\ -2$; $-1,\ -3,\ 2$

-3	-2	-1	1	2	3

$+\ -\ +\ -\ +\ -\ +$

$D_f:(-\infty,-3)\cup[-2,-1)\cup[1,2)\cup[3,+\infty)$

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7) $y=\sqrt{[-x]-2} \rightarrow [-x]-2\geq 0 \rightarrow [-x]\geq 2$

$D_f:(-\infty,-1)$

$y=\dfrac{5x^2+6}{[x]^2-2[x]-3} \rightarrow [x]^2-2[x]-3\neq 0 \rightarrow ([x]-3)([x]+1)\neq 0$

$[x]\neq 3,\ [x]\neq -1 \quad D_f:R-[3,4)\cup[-1,0)$

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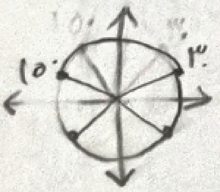
8) $y=\dfrac{\cot x+1}{\tan x+1} \rightarrow \cos x\neq 0,\ \sin x\neq 0,\ \tan x\neq -1$

$D_f=R-\{k\pi,\ k\pi+\frac{\pi}{2},\ 2k\pi+\frac{3\pi}{4},\ 2k\pi+\frac{7\pi}{4}\}$

$D_f=R-\{\frac{k\pi}{2},\ k\pi-\frac{\pi}{4}\}$

$y=\sqrt{1-4\sin^2 x} \quad 1-4\sin^2 x\geq 0 \quad 4\sin^2 x\leq 1 \rightarrow \sin^2 x\leq \frac{1}{4} \quad -\frac{1}{2}\leq \sin x\leq \frac{1}{2}$



$D_f:\{k\pi-\frac{\pi}{6},\ k\pi+\frac{\pi}{6}\}$

$\{2k\pi-\frac{\pi}{6},\ 2k\pi+\frac{\pi}{6}\}\cup\{2k\pi+\frac{5\pi}{6},\ 2k\pi+\frac{7\pi}{6}\}$

۹)

الف) $g = \sqrt{1 - \log_{\frac{1}{2}}^{x-1}}$

$1 - \log_{\frac{1}{2}}^{x-1} \geq 0 \quad , \quad \log_{\frac{1}{2}}^{x-1} \leq 1 \quad , \quad x-1 \leq \frac{1}{2} \rightarrow x \leq \frac{3}{2}$

$D_f \left[\frac{1}{2}, \frac{3}{2}\right]$

ب) $h = \frac{\sqrt{x^2 - x}}{1 - \log_{x^2-3x}^{x}}$ $\rightarrow$ $x^2 - x \geq 0 \quad x(x-1) \geq 0$

$1 \neq \log_{x^2-3x}^{x} \rightarrow x^2 - 3x \neq x \rightarrow x^2 - 4x \neq 0 \quad x(x-4) \neq 0 \quad x \neq 0, 4$

$(x+1)(x-3) \neq 0 \quad x \neq -1, 3$

$(-\infty, 0] \cup [3, +\infty) - \{-1, 3, 4\}$

$y = \sqrt{2^{4x-x^2} - 1}$ $\rightarrow$ $2^{4x-x^2} - 1 \geq 0 \rightarrow 2^{4x-x^2} \geq 1 \rightarrow 2^{4x-x^2} \geq 2^0$

$4x - x^2 \geq 0 \rightarrow -x^2 + 4x \geq 0 \quad , \quad x^2 - 4x \leq 0 \quad x(x-4) \leq 0$

$D_f = [0, 4]$

$y = \left(\frac{3x+5}{2x+4}\right)^{\frac{1}{3}}$ $\rightarrow$ $\frac{3x+5}{2x+4} \in W \quad 3x+5 = 2Wx + 4W \rightarrow 3x - 2Wx = 4W - 5$

$x(3 - 2W) = 4W - 5 \rightarrow x = \frac{4W-5}{3-2W}$

$D_f = \left\{x \mid x = \frac{4k-5}{3-2k}, k \in W\right\}$

$\left(\frac{3x+5}{2x+4}\right)^{\frac{1}{3}} \xrightarrow{W} \frac{4W-5}{3-2W}$