تمرین ها — حل المسائل

۱- $2x^2+y^2-4x+6y+k=0 \;\xrightarrow{k=11=2+9}\; 2x^2-4x+2+y^2+6y+9=0$

$2(x^2-2x+1)$ $\boxed{k=11}$

$x=1 \quad y=-3 \quad \left|\begin{matrix}1\\-3\end{matrix}\right. \qquad (x-1)^2+(y+3)^2=0$

۲- $a^2+4a=2a+a+2 \to a^2+a-2=0 \to (a+2)(a-1)=0$ $\boxed{5}$

$a=1$, $\cancel{a=-2}$, $a>0 \longrightarrow a=1$

$f(1)= \quad x^2+4x \qquad x\geq 1 \to 1+4=\boxed{5}$

۳- $2x^2-b \qquad |x+1|\geq 2 \to 2\leq x+1 \;\cup\; x+1\leq -2$

$a+2x \qquad -3\leq x\leq -1 \qquad 1\leq x \;\cup\; x\leq -3$

$x=-3 \longrightarrow 18-b=a-6 \to a+b=24$

۴- $y=\sqrt{6x-a}+\sqrt{b-3x} \qquad \frac{b}{a}=?$

$b-3x\geq 0 \to x\leq \frac{b}{3}$

$6x-a\geq 0 \to x\geq \frac{a}{6} \qquad \frac{a}{6}\leq x\leq \frac{b}{3} \longrightarrow \frac{a}{6}=\frac{b}{3}=2$

$a=12,\; b=6 \qquad \frac{b}{a}=\frac{6}{12}=\frac{1}{2}$

۵-

الف) $y=\sqrt{\dfrac{x+1}{|x-2|}}+\sqrt{\dfrac{6-x}{x^2+4x+4}}$ $\qquad x=\emptyset$

$(x+2)^2$

$\dfrac{x+1}{|x-2|}\geq 0 \to -1\leq x \qquad \dfrac{6-x}{(x+2)^2}\geq 0 \qquad x\leq 6, \; x\neq -2$

همواره $+$ $\quad x\neq 2 \qquad \cap \qquad$ همواره $+$

$D_y=[-1,6]-\{-2,2\}$

ب) $y=\sqrt{[x]-4}+\sqrt{3-[x]} \qquad [x]-4\geq 0 \to 4\leq [x]$

$3-[x]\geq 0 \to [x]\leq 3 \to x=(-\infty,4) \quad \cap \quad x=[4,+\infty)$

$x=\emptyset$

نقطه‌ها

هلیا معزیان

۶- الف) $y = \sqrt{\dfrac{x^2 - x - 6}{x^4 - 13x^2 + 36}} = \sqrt{\dfrac{(x-3)(x+2)}{(x-2)(x+2)\,(x-3)(x+3)}} \geq 0$

$(x^2-4)(x^2-9)$

$$\begin{array}{c|ccccccccc} x & & -3 & & -2 & & 2 & & 3 & \\ \hline & + & \text{تن} & - & \text{تن} & - & \text{تن} & + & \text{تن} & + \end{array}$$

$D_y = (-\infty, -3) \cup (2, +\infty) - \{3\}$

ب) $y = \sqrt{\dfrac{x^3 - 2x^2 - 5x + 6}{x^3 + 2x^2 - 5x - 6}}$

مجموع ضرائب $= 0 \longrightarrow (x-1)$

$(x+1) \longrightarrow$ جمع ضرائب توان‌های فرد $=$ جمع ضرائب توان‌های زوج

$$\begin{array}{l|l} x^3 - 2x^2 - 5x + 6 & x - 1 \\ \hline -(x^3 - x^2) & x^2 - x - 6 \\ \quad -x^2 - 5x & \\ \quad -(-x^2 + x) & \\ \qquad -6x + 6 & \\ \qquad -(-6x + 6) & \\ \qquad\quad 0 & \end{array}$$

$$\begin{array}{l|l} x^3 + 2x^2 - 5x - 6 & x + 1 \\ \hline -(x^3 + x^2) & x^2 + x - 6 \\ \quad x^2 - 5x & \\ \quad -(x^2 + x) & \\ \qquad -6x - 6 & \\ \qquad -(-6x - 6) & \\ \qquad\quad 0 & \end{array}$$

$y = \sqrt{\dfrac{(x-3)(x+2)}{(x+3)(x-2)}}$

$$\begin{array}{c|ccccccccc} x & & -3 & & -2 & & 2 & & 3 & \\ \hline & + & \text{تن} & - & 0 & + & \text{تن} & - & 0 & + \end{array}$$

$D_y = (-\infty, -3) \cup [-2, 2) \cup [3, +\infty)$

۷- الف) $y = \sqrt{[-x] - 3}$ $\qquad [-x] - 3 \geq 0$

$[-x] \geq 3 \longrightarrow x \in (-\infty, -3]$

$D_y = (-\infty, -3]$

ب) $y = \dfrac{5x^2 + 6}{[x]^2 - 3[x] - 4}$ $\qquad 5x^2+6 \longrightarrow$ همواره $+$

$([x] - 4)([x] + 1)$

$$\begin{array}{c|ccccccc} [x] & & -1 & & 0 & & 4 & \\ \hline & + & \text{تن} & - & & - & \text{تن} & + \end{array}$$

$D_y = (-\infty, -1) \cup [4, +\infty) \cup \{0\}$

حل تمرین

صفحه

۸- الف) $y = \dfrac{\cot x + 1}{\tan x + 1}$ $\quad \cot = \dfrac{\cos}{\sin} \neq 0 \to \sin \neq k\pi$

$\tan + 1 \neq 0 \to \tan \neq -1$ $\quad \tan = \dfrac{\sin}{\cos} \neq 0 \to \cos \neq k\pi + \dfrac{\pi}{2}$

$\tan \neq k\pi - \dfrac{\pi}{4}$

$D_y = \mathbb{R} - \{k\pi, k\pi + \dfrac{\pi}{2}, k\pi - \dfrac{\pi}{4}\}$

ب) $y = \sqrt{1 - 4\sin^2 x}$ $\quad 1 - 4\sin^2 x \geq 0$

$\sin^2 x \leq \dfrac{1}{4} \to -\dfrac{1}{2} \leq \sin x \leq \dfrac{1}{2}$

$D_y = [2k\pi + \dfrac{5\pi}{6}, 2k\pi + \dfrac{\pi}{6}]$ $\quad 2k\pi + \dfrac{5\pi}{6} \leq x \leq 2k\pi + \dfrac{\pi}{6}$

۹- الف) $y = \sqrt{1 - \log_{\frac{1}{2}}^{x-1}}$ $\quad x - 1 > 0 \to x > 1$

$1 - \log_{\frac{1}{2}}^{x-1} \geq 0 \to \log_{\frac{1}{2}}^{x-1} \leq 1 \to x - 1 \leq \dfrac{1}{2} \to x \leq \dfrac{3}{2}$

$D_y = (1, \dfrac{3}{2}]$

ب) $y = \dfrac{\sqrt{x^2 - x}}{1 - \log_4^{x^2 - 3x}} \to x^2 - x \geq 0 \to x(x-1) \geq 0$ $\quad x = (-\infty, 0] \cup [1, +\infty)$

$\to x^2 - 3x > 0$

$x(x-3) > 0$ $\quad x = (-\infty, 0) \cup (3, +\infty)$

$1 - \log_4^{x^2 - 3x} \neq 0 \to \log_4^{x^2 - 3x} \neq 1 \to x^2 - 3x \neq 0$

$x(x-3) \neq 0 \to x \neq 0, 3$

$D_y = (-\infty, 0) \cup (3, +\infty)$

۱۰- الف) $y = \sqrt{2^{4 - x^2} - 8}$ $\quad 2^{4-x^2} - 8 \geq 0 \to 2^{4-x^2} \geq 2^3$

$4 - x^2 \geq 3 \to x^2 \leq 1$

$x = [-1, 1]$

ن – [illegible]

$$\frac{ax+b}{cx+d} = y \rightarrow x = -\frac{dy-b}{cy-a}$$

ج) $y = \left(\frac{3x+5}{2x+4}\right)!$

$$\frac{3x+5}{2x+4} \in W \rightarrow \frac{-4w+5}{2w-3} = x$$

$$D_y = \left\{x \mid x = \frac{-4k+5}{2k-3} ,\ k \in W\right\}$$