DATE / / Subject: ثابت تکلیف ۲ لیانا ابراهیمی

۲۰

۱- $2x^2+y^2-4x+6y+k=0$

$k=0 \qquad 2x^2-4x+k_1 \rightarrow k_1=2$

$(\sqrt{2}x-\sqrt{2})^2 \rightarrow 2x^2-4x+2$

$y^2+6y+k_2 \rightarrow (y+3)^2 \Rightarrow k_2=9$

$y^2+6y+9 \qquad k_1+k_2=11$

۲- $f(x)\begin{cases} x^2+4x & x\geq a \\ & x\geq 1 \\ 2x+a+2 & x<a \\ 2x+3 & x\leq 1\end{cases} \xrightarrow{fa=a}$

$a^2+4a=2a+a+2$

$f(1)=? \qquad (1)^2+4(1)=5$

$2(1)+3=5$

$a^2+4a=3a+2$

$a^2+4a-3a-2=0$

$a^2+a-2=0 \rightarrow a>0$

$(a+2)(a-1)=0 \quad a=-2$ ✗ غ

$a=1$

۳- $f(x)=\begin{cases} 2x^2-b, & |x+1|\geq 2 \\ a+2x, & -3<x\leq 1\end{cases}$

$a+b=?$

$x+1\geq 2 \rightarrow x\geq 1$

$x+1\leq -2 \rightarrow x\leq -3$

$\xrightarrow{\text{اگر } x=-3} 2(-3)^2-b=a+2(-3)$

$18-b=a-6$

$a+b=18+6=24$

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٤)

$y=\sqrt{4x-a}+\sqrt{b-3x}$

$D_y=\{3\}$

$\frac{b}{a}=?$

$12-a=0 \quad a=12$

$b-9=0 \quad b=9$

$\frac{b}{a}=\frac{9}{12}=\frac{3}{4}$

۵)

الف) $y=\sqrt{\frac{x+1}{|x-3|}}+\frac{\sqrt{6-x}}{\sqrt{x^2+2x+4}}$

$x+1 \to x=-1$, $6-x \to x=6$

$|x-3| \to x=3$

همواره مثبت است

$$\begin{array}{c|c|c} & 1 & 3^* \\ - & + & + \end{array} \qquad \begin{array}{c|c} & 6 \\ + & - \end{array}$$

$-1 \quad 3 \quad 6$

$D_f=[-1,3)\cup(3,6]$

ب) $y=\sqrt{[x]-4}+\sqrt{2-[x]}$

$[x]\geq 4 \to x\geq 4$

$2-[x]\geq 0 \to [x]\leq 2 \to x<3$

$3 \quad 4$

$D_f=\varnothing$

۶)

الف) $y=\sqrt{\frac{x^2-x-6}{x^4-13x^2+36}}$

$x^2-x-6 \to (x-3)(x+2)$

$x=3$

$x=-2$

$x^4-13x^2+36 \to (x^2-4)(x^2-9) \to x=\pm 3$

$x=\pm 2$

$$\begin{array}{c|c|c|c|c} & -3 & -2^* & 2 & 3^* \\ + & - & - & + & + \end{array}$$

$D_f=(-\infty,-3)\cup(2,3)\cup(3,+\infty)$

Arman

$(x-1)(x-3)(x+2) \rightarrow x = +1, 3, -2$

ب) $y = \sqrt{\dfrac{x^3 - 2x^2 - 5x + 6}{x^3 + 2x^2 - 5x - 6}}$

$$\begin{array}{l|l} x^3 - 2x^2 - 5x + 6 & x - 1 \\ \hline -x^3 + x^2 & x^2 - x - 6 \\ -x^2 - 5x + 6 & \\ +x^2 - x & \\ -6x + 6 & \\ +6x - 6 & \\ 0 & \end{array}$$

$(x+1)(x+3)(x-2) \quad x = -1, -3, 2$

$$\begin{array}{l|l} x^3 + 2x^2 - 5x - 6 & x + 1 \\ \hline -x^3 - x^2 & x^2 + x - 6 \\ x^2 - 5x - 6 & \\ -x^2 - x & \\ -6x - 6 & \\ +6x + 6 & \end{array}$$

-3	-2	-1	1	2	3
$+$ ∄ $-$	0 $+$	∄ $-$	0 $+$	∄ $-$	0 $+$

$D_f = (-\infty, -3) \cup [-2, -1) \cup [1, 2) \cup [3, +\infty)$

۷ـ

الف) $y = \sqrt{[-x] - 2}$

$[-x] - 2 \geq 0$

$[-x] \geq 2 \rightarrow -x \geq 2$

$x \leq -2$

$D_f = (-\infty, -2]$

ب) $y = \dfrac{5x^2 + 4x}{[x]^2 - 2[x] - 3}$

$([x] - 3)([x] + 1)$

$[x]^2 - 2[x] - 3 \neq 0 \quad [x] \neq 3 \quad [x] \neq -1$

$3 \leq x < 4 \qquad -1 \leq x < 0$

$-1 \quad 0 \quad 3 \quad 4$

$D_f = (-\infty, -1) \cup [0, 3) \cup [4, +\infty)$

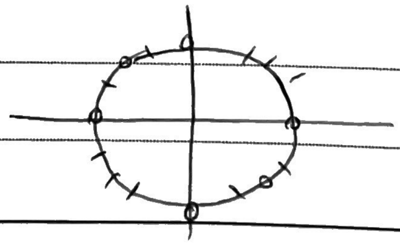
۸

الف) $y = \dfrac{\cot x + 1}{\tan x + 1}$

$\dfrac{\cos}{\sin} \Rightarrow \sin \neq 0$

$\tan x \neq -1 \Rightarrow \cos \neq 0$

$D_f = R - \{\dfrac{k\pi}{2}, k\pi + \dfrac{3\pi}{4}\}$



ب) $y=\sqrt{1-4\sin^2 x}$

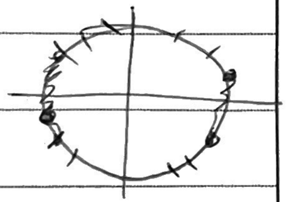
$1-4\sin^2 x \geq 0$

$4\sin^2 x \leq 1$

$\sin^2 x \leq \frac{1}{4} \Rightarrow -\frac{1}{2} \leq \sin x \leq \frac{1}{2}$

$[k\pi-\frac{\pi}{6}, k\pi+\frac{\pi}{6}]$

$[2k\pi+\frac{5\pi}{6}, 2k\pi+\frac{7\pi}{6}] \cup [2k\pi-\frac{\pi}{6}, 2k\pi+\frac{\pi}{6}]$



الف) $y=\sqrt{1-\log_{\frac{1}{2}}^{x-1}}$ $\qquad 1-\log_{\frac{1}{2}}^{x-1} \geq 0$ $\qquad -9$

$x-1>0 \to x>1$ $\qquad 1 \quad \frac{3}{2}$

$\log_{\frac{1}{2}}^{x-1} \leq 1 \to x-1 \geq \frac{1}{2} \to x \geq \frac{3}{2}$ $\qquad [\frac{3}{2}, +\infty)$

ب) $y=\dfrac{\sqrt{x^2-x}}{1-\log_4^{x^2-3x}}$ $\qquad x^2-x \geq 0$ $\qquad$ 0 1, + − +

$x(x-1) \geq 0$

$x^2-3x>0$ $\qquad$ 0 3

$x(x-3)$ $\qquad$ + − +

$(-\infty, 0) \cup (3, +\infty) - \{-1, 4\}$

$1-\log_4^{x^2-3x} \neq 0 \qquad \log_4^{x^2-3x} \neq 1$

$x^2-3x \neq 4 \Rightarrow x \neq 4, -1$

$x^2-3x-4 \neq 0 \to (x-4)(x+1)=0$

الف) $\sqrt{2^{4x-x^2}-8}$ $\quad 2^{4x-x^2}-8 \geq 0$ حل:

$2^{4x-x^2} \geq 2^3 \to 4x-x^2-3 \geq 0$ $\quad$ $1 \quad 3$: $+ \quad - \quad +$

$x^2-4x+3 \leq 0$

$(x-3)(x-1) \leq 0$ $\quad [1,3]$

ب) $y=\left(\frac{3x+5}{2x+4}\right)!$ $\quad \frac{3x+5}{2x+4} \in W$

$3x+5 \in 2xW+4W$

$3x-2xW \in 4W-5$

$x(3-2W) \in 4W-5 \Rightarrow x \in \frac{4W-5}{3-2W}$

$\{x \mid x=\frac{4k-5}{3-2k}, k \in W\}$