19,0

①

$$2x^2 - 8x + 5 + y^2 + 4y + 4 = 0$$

$$x^2 - 2x + x^2 - 2x + 2 + (y+2)^2 = 0$$

$$(x-1)^2 + (x-1)^2 + (y+2)^2 = 0$$

$x = 1$, $y = -2$

$k = 2 + 9 = 11$

✓ (5)

③

$$f(x) \begin{cases} x^2 + 4x & x \geq a \\ 2x + a + 2 & x < a \end{cases}$$

$x = a$ $\quad a^2 + 4a = 2a + a + 2$

$$a^2 + a - 2 = 0 \rightarrow (a+2)(a-1) = 0$$

$a = -2$, $a = 1$

✓ (5)

$a \neq 0$

$\Rightarrow a = 1 \rightarrow f(x) \begin{cases} x^2 + 4x & x \geq 1 \\ 2x + 3 & x < 1 \end{cases}$

$x = 1 \Rightarrow (1)^2 + 4(1) = 5$

④

$a + b =$

$$f(x) \begin{cases} 2x^2 - b & x \geq -1 \\ a + 2x & x < -1 \end{cases}$$

$2 \leq x + 1 \Rightarrow 1 \leq x$

$x + 1 \leq 2$

$x \leq 1$

$x = -2 \Rightarrow 2 \times (-2)^2 - b = a + 2 \times (-2)$

$8 - b = a - 4 \Rightarrow a + b = 12$ ✓

⑤

$$y = \sqrt{4x - a} + \sqrt{b - 2x}$$

$\frac{b}{a}$

$\left(\frac{b}{2}\right)^2 = \left(\frac{a}{4}\right)^{12} - 2$

$4x - a \geq 0$

$4x \geq a$

$x \geq \frac{a}{4}$ ✓ (3)

$D_f = \{2\}$

$\Rightarrow \frac{b}{a} = \frac{4}{12} = \frac{1}{3}$

$b - 4x \geq 0 \Rightarrow b \geq 2x \Rightarrow \frac{b}{2} \leq x$

بنام خداوند بخشنده

۵) الف) $y=\sqrt{\frac{x+1}{|x-3|}}+\sqrt{\frac{6-x}{x^2+2x+3}}$

$x+1\geq 0 \Rightarrow x\geq -1$ (اختیار ④)

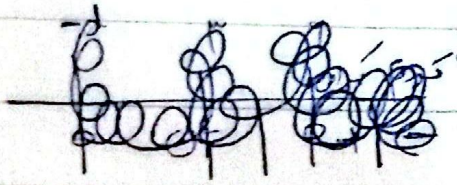
$|x-3|=0 \Rightarrow x=3$

$x+1=0 \Rightarrow x=-1$

$x^2+2x+3>0$ (همیشه)

$6-x\geq 0 \Rightarrow x\leq 6$

$\Delta=b^2-4ac=4-4\times 1\times 3=-12<0$



$D_f=[-1,3)\cup(3,6]$

ب) $y=\sqrt{[x]-4}+\sqrt{2-[x]}$

$[x]-4\geq 0 \Rightarrow [x]\geq 4 \Rightarrow x\geq 4$

$2-[x]\geq 0 \Rightarrow [x]\leq 2 \Rightarrow x<3$

$D_f=\emptyset$

۶) الف) $y=\sqrt{\frac{x^2-x-6}{x^4+3x^3-4x^2}}$

$x^2-x-6=0 \Rightarrow (x-3)(x+2)=0 \Rightarrow x=3,-2$

$x^4+3x^3-4x^2=0 \Rightarrow x^2(x^2+3x-4)=0$

$t^2+3t-4=0 \Rightarrow (t+4)(t-1)=0$

$t=-4 \Rightarrow x^2=-4$ (غ ق ق)

$t=1 \Rightarrow x^2=1 \Rightarrow x=\pm 1$

به نام خداوند دانش

$$\frac{(x-3)(x+2)}{(x+3)(x-3)(x+2)(x-2)}$$ ۲/۵

-3	-2^*	2	3^*	
$+$	$-$	$-$	$+$	$+$

$D_f = (-\infty, -3) \cup (2, 3) \cup (3, +\infty)$

ب) $y = \sqrt{\dfrac{x^3 - 2x^2 - 5x + 6}{x^3 + 2x^2 - 5x - 6}}$ ⇐ جمع ضرایب = ۰ ⇐ بخش پذیر بر $(x-1)$

⇐ جمع ضرایب توان های زوج با فرد برابری توان های ⇐ بخش پذیر بر $(x+1)$

$$\begin{array}{l} x^3 - 2x^2 - 5x + 6 \quad \big| \; x-1 \\ \ominus x^3 \oplus x^2 \qquad\qquad\quad x^2 - x - 6 \\ \hline -x^2 - 5x \\ -x^2 + x \\ \oplus \quad \ominus \\ \hline -6x + 6 \\ -6x + 6 \\ \hline 0 \end{array}$$

$$\begin{array}{l} x^3 + 2x^2 - 5x - 6 \quad \big| \; x+1 \\ \underline{-}x^3 \underline{-}x^2 \qquad\qquad\quad x^2 + x - 2 \\ \hline x^2 - 5x \\ \ominus x^2 \ominus x \\ \hline -6x - 6 \\ -6x - 6 \\ \hline 0 \end{array}$$

$$y = \sqrt{\frac{(x-1)(x-3)(x+2)}{(x+1)(x+3)(x-2)}}$$

ریشه‌های صورت: 1 ، 3 ، -2 — ریشه‌های مخرج: -1 ، -3 ، 2

-3	-2	-1	1	2	3	
$+$	$-$	$+$	$-$	$+$	$-$	$+$

۶

$D_f = (-\infty, -3) \cup [-2, -1) \cup [1, 2)$
$\cup \; [3, +\infty)$

بنام خداوند دانش

(۷)

الف) $y=\sqrt{[-x]-2}$

$[-x]-2\geq 0 \Rightarrow [-x]\geq 2 \Rightarrow -x\geq 2$

$x\leq -2$

$D_f=(-\infty , -2]$

ب) $y=\dfrac{5x^2+4}{[x]^2-2[x]-3}$

$[x]^2-2[x]-3\neq 0$

$([x]-3)([x]+1)\neq 0$

$[x]\neq 3 \ / \ [x]\neq -1$

$\Downarrow \qquad \Downarrow$

$3\leq x<4 \qquad -1\leq x<0$

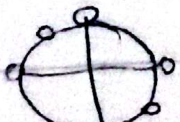
$D_f=\mathbb{R}-[3,4)-[-1,0)$

(۸)

الف) $y=\dfrac{\cot x+1}{\tan x+1} \Rightarrow \tan x+1\neq 0 \Rightarrow \tan x\neq -1$

$\cot x=\dfrac{\cos x}{\sin x} \Rightarrow \neq 0$

$\tan x=\dfrac{\sin x}{\cos x} \Rightarrow \neq 0$



$D_f=\mathbb{R}-\left\{\dfrac{k\pi}{2} , k\pi-\dfrac{\pi}{4}\right\}$

ب) $y=\sqrt{1-4\sin^2 x}$

$-\frac{1}{2}\leq \sin x\leq \frac{1}{2}$

$(1+2\sin x)(1-2\sin x)=0$

$1-4\sin^2 x\geq 0$

$1\geq 4\sin^2 x \Rightarrow \frac{1}{4}\geq \sin^2 x$

بنام خداوند دانش

$D_f = [k\pi - \frac{\pi}{4} , k\pi + \frac{\pi}{4}]$ ۲

(۹) ۱,۶

الف) $y = \sqrt{1 - \log_{\frac{1}{2}}^{x-1}}$ $x - 1 > 0 \Rightarrow x > 1$

$1 - \log_{\frac{1}{2}}^{x-1} \geq 0 \Rightarrow 1 \geq \log_{\frac{1}{2}}^{x-1}$

$D_f = (1 , \frac{3}{2}]$ $\frac{1}{2} \geq x - 1 \Rightarrow \frac{3}{2} \geq x$

(blue correction: $\leq$) $D_f = [\frac{3}{2} , +\infty)$

ب) $y = \frac{\sqrt{x^2 - x}}{1 - \log_{4}^{x^2 - 3x}}$ $x(x-1) \geq 0$

0 1

$+ \quad - \quad +$

$x^2 - 3x > 0 \Rightarrow x(x-3) > 0$ $\cap$

0 3

0 3

$+ \quad - \quad +$

$1 - \log_{4}^{x^2 - 3x} \neq 0$

$1 \neq \log_{4}^{x^2 - 3x}$

$4 \neq x^2 - 3x \Rightarrow x^2 - 3x - 4 \neq 0$

$(x - 4)(x + 1) \neq 0$

4 -1

$(-\infty , 0) \cup (3 , +\infty)$

$\Rightarrow D_f = (-\infty , 0) \cup (3 , +\infty) - \{4 , -1\}$

(۱۰)

الف) $y = \sqrt{2^{4x - x^2} - 8}$

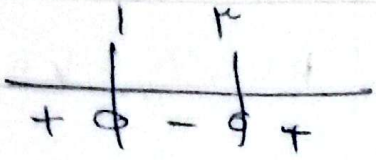
$2^{4x - x^2} - 8 \geq 0 \Rightarrow 2^{4x - x^2} \geq 8$

3 1 $x - 1 = 0 \Rightarrow x = 1$ $4x - x^2 \geq 3$

$x - 3 = 0 \Rightarrow x = 3$

$\Rightarrow (x - 3)(x - 1) \leq 0$ $0 \geq x^2 - 4x + 3$

بنابراضافه شد



$D_f = [1,4]$

ب) $y = \left(\frac{4x+5}{2x+4}\right)!$

«تعریف نشده»

$\frac{4x+5}{2x+4} \in \mathbb{W} \Rightarrow 4x+5 \in 2x+4$

$2x+4w \in 4x+5$

$2xw - 4x \in 5-4w$

$x(2w-4) \in 5-4w$

$x \in \frac{5-4w}{2w-4}$

$D_f = \{x \mid x = \frac{5-4k}{2k-4}, k \in \mathbb{W}\}$