سارینا اسپنانی - یازدهم ریاضی - شماره تکلیف ۲

① $2x^2+y^2-4x+6y+k=0$

$(\sqrt{2}x-2)^2+(y+3)^2=0$

$2x^2-4x+4+y^2+9+6y=0$

$k=4+9=11$

② $f(x)=\begin{cases} x^2+4x & x\geq a \\ 3x+a+2 & x\leq a \end{cases}$ $\quad f(1)=?$ $\quad a>0$

$a^2+4a=3a+a+2$

$a^2+4a-3a-2=0 \rightarrow a^2+a-2=0 \rightarrow (a+2)(a-1)=0$

$f(1)=1+4=2+1+2=5$ $\quad \boxed{a=-2} \rightarrow$ غ ق ق $\quad \boxed{a=1} \rightarrow a>0 \rightarrow$ ق ق

③ $f(x)=\begin{cases} 2x^2-b & |x+1|\geq 2 \\ a+2x & -3\leq x<-1 \end{cases}$ $\quad a+b=?$

$|x+1|\geq 2 \longrightarrow 1\leq x \,,\; x\leq -3$ (بازای -3 برابر می‌شوند)

$2(-3)^2-b=a+2(-3)$

$18+6=a+b=24$

④

$y = \sqrt{6x - a} + \sqrt{b - 3x}$ $\qquad \frac{b}{a} = ? \rightarrow \frac{6}{12} = \frac{1}{2}$

$6x - a \geq 0 \rightarrow 6x \geq a \rightarrow 12 \geq a \rightarrow \boxed{a = 12}$

$b - 3x \geq 0 \rightarrow 3x \leq b \rightarrow 6 \leq b \rightarrow \boxed{b = 6}$

چون دامنه فقط به ازای ۲ پاسخ می‌دهد پس زیر رادیکال‌ها برابر با صفر می‌شود.

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⑤

الف) $y = \sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{6-x}{x^2+2x+4}}$

ریشه‌ها: -1 ، $*(+3)$ مضاعف $\qquad$ ریشه‌ها: 6 $\qquad x^2+2x+4=0$

-1		3^*	
$-$	$+$	ت ن	$+$

6	
$+$	$-$

$\Delta = b^2 - 4ac = 4 - 4(4)(1) = -12$ ریشه ندارد

$(II) \rightarrow [-1, 3) \cup (3, +\infty) \qquad (I) \Rightarrow (-\infty, 6]$

$D_f = (I) \cap (II) \rightarrow [-1, 3) \cup (3, 6]$

ب) $y = \sqrt{[x] - 4} + \sqrt{2 - [x]}$

$[x] - 4 \geq 0 \rightarrow [x] \geq 4 \rightarrow x \geq 4 \quad (I)$

$2 - [x] \geq 0 \rightarrow [x] \leq 2 \rightarrow x < 3 \quad (II)$

$D_f = (I) \cap (II) \rightarrow \varnothing$

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⑥

الف) $\sqrt{\frac{x^2 - x - 6}{x^4 - 13x^2 + 36}}$

$\frac{x^2 - x - 6}{x^4 - 13x^2 + 36} \geq 0 \qquad \frac{(x-3)(x+2)}{(x-2)(x+2)(x-3)(x+3)} \geq 0$

-3	-2^*	2	3^*	
$+$ ت ن	$-$ ت ن	$-$ ت ن	$+$ ت ن	$+$

$D_f = (-\infty, -3) \cup (2, 3) \cup (3, +\infty)$

ب) $\sqrt{\dfrac{x^3-2x^2-5x+6}{x^3+2x^2-5x-6}}$

$$\frac{\overbrace{x^3-2x^2-5x+6}^{(x^2-x-6)(x-1)}}{\underbrace{x^3+2x^2-5x-6}_{(x^2+x-6)(x+1)}} \geq 0 \qquad \frac{\overbrace{(x-3)}^{3}\overbrace{(x+2)}^{-2}\overbrace{(x-1)}^{1}}{\underbrace{(x+3)}_{-3}\underbrace{(x-2)}_{2}\underbrace{(x+1)}_{-1}} \geq 0$$

	-3		-2		-1		1		2		3	
+	ن ت	−	0	+	ن ت	−	0	+	ن ت	−	0	+

$D_f = (-\infty,-3)\cup[-2,-1)\cup[1,2)\cup[3,+\infty)$

الف) $y=\sqrt{[-x]-2}$ (7)

$[-x]-2 \geq 0 \longrightarrow [-x]\geq 2$

$-x \geq 2 \longrightarrow \boxed{x\leq -2}$

$D_f = (-\infty,-2]$

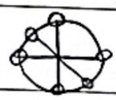
ب) $y=\dfrac{5x^2+6}{[x]^2-2[x]-3}$

$[x]^2-2[x]-3 \neq 0 \longrightarrow ([x]-3)([x]+1)\neq 0$

$[x]\neq 3 \qquad [x]\neq -1$

$D_f = \mathbb{R} - [-1,0)\cup[3,4)$

الف) $y=\dfrac{\cot x+1}{\tan x+1}$ (8)



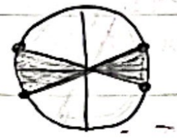
(I) $\tan x+1\neq 0 \longrightarrow \tan x\neq -1$

(II) $\tan x=\dfrac{\sin x}{\cos x} \qquad \cos x\neq 0$

(III) $\cot x=\dfrac{\cos x}{\sin x} \qquad \sin x\neq 0$

$(I)\cap(II)\cap(III) \longrightarrow D_f=\mathbb{R}-\left\{\dfrac{k\pi}{2},\ k\pi-\dfrac{\pi}{4}\right\}$

ب) $y=\sqrt{1-4\sin^2 x}$



$1-4\sin^2 x \geqslant 0$

$1 \geqslant 4\sin^2 x$

$\sin^2 x \leqslant \frac{1}{4}$ $\quad\quad -\frac{1}{2} \leqslant \sin x \leqslant \frac{1}{2}$

$D_f=\left[k\pi-\frac{\pi}{6}, k\pi+\frac{\pi}{6}\right]$

الف) $y=\sqrt{1-\log_{\frac{1}{2}}^{x-1}}$ ۹

$x-1>0 \rightarrow \boxed{x>1}$ (I)

$1-\log_{\frac{1}{2}}^{x-1} \geqslant 0$

$\log_{\frac{1}{2}}^{x-1} \leqslant 1$ $\quad\quad (I)\cap(II) \rightarrow D_f=\left(1, \frac{3}{2}\right]$

$x-1 \leqslant \frac{1}{2}$ $\quad \boxed{x \leqslant \frac{3}{2}}$ (II)

ب) $y=\frac{\sqrt{x^2-x}}{1-\log_4^{x^2-3x}}$

$x^2-x \geqslant 0 \rightarrow x(x-1) \geqslant 0$

$(I) \rightarrow (-\infty, 0]\cup[1, +\infty)$ $\quad \begin{array}{ccccc} & 0 & & 1 & \\ + & \phi & - & \phi & + \end{array}$

$1-\log_4^{x^2-3x} \neq 0 \rightarrow 1 \neq \log_4^{x^2-3x}$

$4 \neq x^2-3x \rightarrow x^2-3x-4 \neq 0$

$x^2-3x>0$ $\quad\quad (x-4)(x+1) \neq 0$

$x(x-3)>0$ $\quad \begin{array}{ccccc} & 0 & & 3 & \\ + & \phi & - & \phi & + \end{array}$ $\quad$ (II) $\boxed{x \neq 4, -1}$

$\underbrace{0 \text{ و } 3}$

$(I \cap I) \rightarrow (-\infty, 0)\cup(3, +\infty)$

$D_f=(-\infty, 0)\cup(3, +\infty)-\{4, -1\}$

الف) $y=\sqrt{2^{4x-x^2}-8}$ (10)

$2^{4x-x^2}-8\geq 0$

$2^{4x-x^2}\geq 8$

$2^{4x-x^2}\geq 2^3 \to 4x-x^2\geq 3 \to 0\geq x^2-4x+3$

$$\begin{array}{c|c|c}
 & 1 & 3 & \\
\hline
+ & - & +
\end{array}$$

$0\geq \underbrace{(x-3)}_{3}\underbrace{(x-1)}_{1}$

$D_f=[1,3]$

ب) $y=\left(\frac{3x+5}{2x+4}\right)!$

$\frac{3x+5}{2x+4}\in W$ $\qquad x=-\frac{4W-5}{2W-3}$

$D_f=\left\{x \mid x=\frac{-4k+5}{2k-3}, k\in W\right\}$

روش تشریحی $\to$ $\frac{3x+5}{2x+4}\in W \to 3x+5\in 2xW+4W$

$3x-2xW\in 4W-5$

$x(3-2W)\in 4W-5$

$x\in\frac{4W-5}{3-2W}=\frac{-4W+5}{2W-3}$