

19,0

سارینا استیانی - یازم ریاضت - شمس و سولید ۷

$$x^2 + y^2 - 4x + 4y + k = 0$$

①

$$(\sqrt{x} - 2)^2 + (y + 2)^2 = 0$$

$$x^2 - 4x + 4 + y^2 + 4y + 4 = 0$$

$$k = -4 + 4 = 0$$

5

$$f(x) = \begin{cases} x^2 + 2x & x \geq a \\ 2a + a + 2 & x \leq a \end{cases}$$

$f(1) = ?$

$a > 0$

۲

$$a^2 + 2a = 2a + a + 2$$

$$a^2 + 2a - 2a - 2 = 0 \rightarrow a^2 + a - 2 = 0 \rightarrow (a+2)(a-1) = 0$$

$$f(1) = 1 + 2 = 2 + 1 + 2 = 5$$

$$\begin{matrix} a = -2 & a = 1 \end{matrix} \rightarrow a > 0$$

$$f(x) = \begin{cases} x^2 - b & |x+1| \geq 2 \\ a + 2x & -3 \leq x \leq -1 \end{cases}$$

$a + b = ?$

۳

$$|x+1| \geq 2 \rightarrow 1 \leq x, \quad x \leq -3 \quad (\text{بزرگتر از ۳ یا کوچکتر از -۳})$$

$$f(-3) - b = a + 2(-3)$$

$$11 + 9 = a + b = 20$$

Scbo

$$y = \sqrt{4a - a} + \sqrt{b - 3a}$$

$$\frac{b}{a} = ? \rightarrow \frac{4}{12} = \left(\frac{1}{3}\right) \quad (4)$$

$$4a - a \geq 0 \rightarrow 4a \geq a \rightarrow 12 \geq a \rightarrow \boxed{a = 12}$$

$$b - 3a \geq 0 \rightarrow 12 \leq b \rightarrow 4 \leq b \rightarrow \boxed{b = 4} \quad (5)$$

چون رابطه فقط به ازای ۲ پاسخ می دهد پس زیر این عمل ها برابر با صفر می شوند.

$$الف) y = \sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{4-x}{2^2+2x+1}}$$

(5)

①: $\frac{x+1}{|x-3|}$ که
 $\begin{array}{c|c|c|c|c|c} * & + & 1 & & * & \\ \hline - & 1 & 3 & & + & \\ \hline - & 1 & 3 & & + & \end{array}$

④: $\frac{4-x}{2^2+2x+1}$ که

$$\begin{array}{c|c|c|c|c|c} & & 4 & & & \\ \hline + & 1 & 3 & & - & \end{array}$$

$$2^2+2x+1 = 0$$

$$\Delta = b^2 - 4ac = 1 - 4(1)(1) = -3$$

کمتر از صفر

$$(II) \rightarrow [-1, 3) \cup (3, +\infty) \quad (I) \Rightarrow (-\infty, 4]$$

$$Df = (I) \cap (II) \rightarrow [-1, 3) \cup (3, 4]$$

$$ب) y = \sqrt{[x] - 4} + \sqrt{4 - [x]}$$

$$[x] - 4 \geq 0 \rightarrow [x] \geq 4 \rightarrow x \geq 4 \quad (I)$$

$$4 - [x] \geq 0 \rightarrow [x] \leq 4 \rightarrow x \leq 4 \quad (II)$$

$$Df = (I) \cap (II) \rightarrow \emptyset$$

$$الف) \sqrt{\frac{x^2 - x - 4}{x^2 - 13x^2 + 34}}$$

(9)

$$\frac{x^2 - x - 4}{x^2 - 13x^2 + 34} \geq 0 \quad \frac{(x-3)(x+2)}{(x-2)(x+2)(x-3)(x+3)} \geq 0$$

$$\begin{array}{c|c|c|c|c|c} * & & * & & & \\ \hline - & 3 & - & 2 & + & 3 \\ \hline + & 0 & - & 0 & - & 0 \end{array}$$

$$Df = (-\infty, -3) \cup (2, 3) \cup (3, +\infty)$$

$$i.) \sqrt{\frac{x^3 - 2x^2 - 2x + 4}{x^3 + 2x^2 - 2x - 4}}$$

$$\begin{array}{ccccccc} -3 & -2 & -1 & 1 & 2 & 3 \\ + & - & + & - & + & - & + \end{array}$$

$$(x^3 - x - 4)(x - 1)$$

$$\frac{x^3 - 2x^2 - 2x + 4}{x^3 + 2x^2 - 2x - 4} \geq 0 \Rightarrow \frac{(x-1)(x+2)(x-1)}{(x+2)(x-2)(x+1)} \geq 0$$

$$Df = (-\infty, -2) \cup [-2, -1) \cup [1, 2) \cup [2, +\infty)$$

$$ii.) y = \sqrt{[-x] - 2}$$

(v)

$$[-x] - 2 \geq 0 \rightarrow [-x] \geq 2$$

$$-x \geq 2 \rightarrow x \leq -2$$

$$Df = (-\infty, -2]$$

$$i.) y = \frac{2x^2 + 4}{[x]^2 - 2[x] - 3}$$

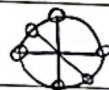
$$[x]^2 - 2[x] - 3 \neq 0 \rightarrow ([x] - 3)([x] + 1) \neq 0$$

$$[x] \neq 3 \quad [x] \neq -1$$

$$Df = \mathbb{R} - [-1, 0) \cup [3, 4)$$

$$ii.) y = \frac{\cot x + 1}{\tan x + 1}$$

(A)



$$(I) \tan x + 1 \neq 0 \rightarrow \tan x \neq -1$$

$$(II) \tan x = \frac{\sin x}{\cos x} \quad \cos x \neq 0$$

$$(III) \cot x = \frac{\cos x}{\sin x} \quad \sin x \neq 0$$

$$(I) \cap (II) \cap (III) \rightarrow Df = \mathbb{R} - \left\{ \frac{k\pi}{4}, k\pi - \frac{\pi}{4} \right\}$$

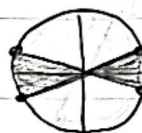
$$b) y = \sqrt{1 - k \sin^2 x}$$

$$1 - k \sin^2 x \geq 0$$

$$1 \geq k \sin^2 x$$

$$\sin^2 x \leq \frac{1}{k}$$

$$-\frac{1}{\sqrt{k}} \leq \sin x \leq \frac{1}{\sqrt{k}}$$



(5)

$$Df = \left[k\pi - \frac{\pi}{4}, k\pi + \frac{\pi}{4} \right]$$

$$iii) y = \sqrt{1 - \log \frac{x-1}{r}}$$

$$x-1 \geq 0 \rightarrow \boxed{x \geq 1} \quad (I)$$

$$1 - \log \frac{x-1}{r} \geq 0$$

$$\log \frac{x-1}{r} \leq 1$$

$$(I) \cap (II) \rightarrow Df = \left(1, \frac{r}{r-1}\right]$$

$$x-1 \leq \frac{1}{r}$$

$$\boxed{x \leq \frac{r}{r-1}} \quad (II)$$

$$Df = \left[\frac{r}{r-1}, +\infty\right)$$

$$b) y = \frac{\sqrt{x^2 - k}}{1 - \log \frac{x^2 - k}{r}}$$

$$x^2 - k \geq 0 \rightarrow x(x-1) \geq 0$$

$$(I) \rightarrow (-\infty, 0] \cup [1, +\infty)$$

$$\frac{0}{+ \phi - \phi +}$$

$$1 - \log \frac{x^2 - k}{r} \neq 0 \rightarrow 1 \neq \log \frac{x^2 - k}{r}$$

$$k \neq x^2 - k \rightarrow x^2 - k - k \neq 0$$

$$x^2 - k \geq 0$$

$$(x-k)(x+1) \neq 0$$

$$x(x-k) \geq 0$$

$$\frac{0}{+ \phi - \phi +}$$

$$(II) \boxed{x \neq k, -1}$$

$$(I \cap II) \rightarrow (-\infty, 0) \cup (k, +\infty)$$

$$Df = (-\infty, 0) \cup (k, +\infty) - \{k, -1\}$$

Scbó

$$الف) y = \sqrt{r^{kn-2r} - \Lambda}$$

$$r^{kn-2r} - \Lambda \geq 0$$

(6)

$$r^{kn-2r} \geq \Lambda$$

$$r^{kn-2r} \geq r^r$$

$$\rightarrow kn-2r \geq r \rightarrow 0 \geq 2r - kn + r$$

$$\begin{array}{c} 1 \quad r \\ + \phi - \phi + \end{array}$$

$$0 \geq \underbrace{(2-r)}_r \underbrace{(n-1)}_1$$

$$Df = [1, r]$$

$$ب) y = \left(\frac{r^{n+\omega}}{r^{n+k}} \right)!$$

(5)

$$\frac{r^{n+\omega}}{r^{n+k}} \in W$$

$$n = -\frac{rW - \omega}{rW - r}$$

$$Df = \left\{ n \mid n = \frac{-rk + \omega}{rk - r}, k \in W \right\}$$

روشن نشود

$$\frac{r^{n+\omega}}{r^{n+k}} \in W, r^{n+\omega} \in r^{nW} + rW$$

$$r^n - r^{nW} \in rW - \omega$$

$$n(r - rW) \in rW - \omega$$

$$n \in \frac{rW - \omega}{r - rW} = \frac{-rW + \omega}{rW - r}$$