۱)

$3x^2 - ax + 4 = 0$ $\qquad 4\beta = \frac{a}{3}$ $\qquad \alpha = 3\beta$

اختلاف دو مقدار a در حالت $\alpha = 3\beta$ ؟

$3\beta \times \beta = 3\beta^2 \qquad 3\beta^2 = \frac{4}{3} \qquad \beta^2 = \frac{4}{9} = \pm\frac{2}{3}$

$4 \times \frac{2}{3} = \frac{a}{3} \rightarrow \frac{8}{3} = \frac{a}{3} \qquad \boxed{a_1 = 8}$ $\qquad$ پاسخ $= a_1 - a_2 = 16$

$\beta = -\frac{2}{3} \Rightarrow \alpha = -2 \qquad -2 - \frac{2}{3} = a \qquad \boxed{a_2 = -8}$

۲)

$\sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = 5$

$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = 5 \qquad \frac{\sqrt{\beta} + \sqrt{\alpha}}{\sqrt{\alpha\beta}} = 5 \qquad \frac{\sqrt{\alpha} + \sqrt{\beta}}{\frac{1}{6}} = 5 \Rightarrow \sqrt{\alpha} + \sqrt{\beta} = \frac{5}{6}$

$\alpha\beta = \frac{1}{36} \qquad \alpha + \beta = \frac{m+14}{36} \qquad \alpha + \beta + 2\sqrt{\frac{1}{36}} = \frac{25}{36}$

$mx^2 + 3x + 2 = 0 \qquad \frac{m+14}{36} = \frac{25}{36} - \frac{12}{36} = \frac{13}{36}$

$-x^2 + 3x + 2 = 0 \qquad \boxed{\frac{c}{a} = -2}$

$m + 14 = 13$

$\boxed{m = -1}$

۳)

$(\alpha + \beta)^2 = 1 \rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 1 \qquad \left.\begin{array}{l} \alpha^2 + \beta^2 = 5 \\ \alpha + \beta = 1 \end{array}\right\} \begin{array}{l} \alpha = 2 \\ \beta = -1 \end{array}$

$4 \times (-1)^3 + k + 9 - 2 = 0 \rightarrow \boxed{k = -3}$ ✓

$4 \times 8 + 4k - 18 - 2 = 0 \Rightarrow \boxed{k = -3}$ ✓

۴

$۳\alpha^۲ + ۲\alpha^۲ = ۲,۵\alpha^۲ + ۲,۵\beta^۲ + ۰,۵\alpha^۲ - ۰,۵\beta^۲$

$\frac{۵}{۲}(\alpha^۲+\beta^۲) + \frac{۱}{۲}(\alpha^۲-\beta^۲)$ $\quad \alpha^۲+\beta^۲ = S^۲-۲P \Rightarrow ۳۶-۲a$

$\alpha^۲-\beta^۲ = (\alpha+\beta)(\alpha-\beta)$

$\Rightarrow -۶(-\sqrt{۳۶-۴a}) = ۶\sqrt{۳۶-۴a}$

$\frac{\sqrt{\Delta}}{|a|} = \alpha<\beta \rightarrow -\sqrt{\Delta}$

$\frac{۵}{۲}(\alpha^۲+\beta^۲) + \frac{۱}{۲}(\alpha^۲-\beta^۲) \Rightarrow ۹۰-۵a+۳\sqrt{۳۶-۴a}$

$\Rightarrow ۱۲\sqrt{۲}+۸۵$

$۹۰-۵a = ۸۵ \rightarrow a=۱$ $\quad ۳\sqrt{۳۶-۴a} = ۱۲\sqrt{۲} \Rightarrow \boxed{a=۱}$ ✓

۵

$x^۲ = t$ $\quad t = \frac{+۷ \pm \sqrt{۴۹}}{۲}$

ق ق

ق غ ق

$t^۲ - ۷t - ۵ = ۰$

$\Delta = ۴۹+۲۰$ $\quad x = \pm\sqrt{\frac{۷+\sqrt{۴۹}}{۲}}$ $\quad S=۰ \rightarrow P = \frac{-۷-\sqrt{۴۹}}{۲}$

$۲P^۲ - ۲P\cancel{S} + \cancel{۲S} = ۰$

$۲P^۲ = \frac{۴۹+۴۹+۱۴\sqrt{۴۹}}{۴} \times ۲ = \boxed{۵۹+۷\sqrt{۴۹}}$

۶

$\alpha+\beta = -\frac{۱}{۲} \quad \alpha\beta = -\frac{۳}{a}$

$\alpha+\beta = -\frac{۱}{۲}$

$\alpha+\beta = \frac{a+(-۲)}{۲} \Rightarrow \boxed{a=۱}$

$(\frac{۱}{۲}+\alpha)(\frac{۱}{۲}+\beta) = \frac{۱}{۴} + (\alpha+\beta)\frac{۱}{۲} + \alpha\beta = \frac{b}{۲}$

$\frac{۱}{۴} - \frac{۱}{۴} - \frac{۱۲}{۴} = \frac{b}{۲} \rightarrow b = -۶$ $\quad [\frac{ab}{۴}] = [\frac{۱\times-۶}{۴}] = [-\frac{۳}{۲}] = -۲$

(۷)

$s=1$
$p=-\frac{b}{a}$

$a\alpha^2-a\alpha-b=0$

$a\beta^2-a\beta-b=0 \rightarrow \beta^2-\beta=\frac{b}{a}$

$40\beta^2+20\alpha^2-20\beta=20\beta^2+20\alpha^2+20\beta^2-20\beta$

$20(s^2-2p)+20(\beta^2-\beta) \Rightarrow 20(1+\frac{2b}{a})+20(\frac{b}{a})=$

$\rightarrow 20+\frac{40b}{a}+\frac{20b}{a}=20+60\frac{b}{a}=17$

$\frac{b}{a}=-\frac{1}{20} \rightarrow b=-\frac{1}{20}a \qquad \frac{\sqrt{\Delta}}{|a|}=\frac{\sqrt{a^2+4ab}}{|a|}$

$\frac{\sqrt{a^2-\frac{a^2}{5}}}{|a|}=\frac{\cancel{|a|}\sqrt{\frac{4}{5}}}{\cancel{|a|}}=\boxed{\frac{2}{\sqrt{5}}}$

پاسخ

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(۸)

$a=2k+1 \times 2k-1=4k^2-1$

$a+1=4k \rightarrow a=4k-1$

$4k^2-4k=0$

$4k(k-1)=0 \rightarrow k=1 \quad k=0$

$\boxed{a=3}$

$b=4k'^2+4k'$

ریشه‌های معادله اول $\rightarrow 2k+1 \,,\, 2k-1$

ریشه‌های معادله دوم $\rightarrow 2k' \,,\, 2k'+2$

$3a+1=4k'+2 \qquad \boxed{|a-b|=21}$

$10=4k'+2 \rightarrow k'=2 \qquad \boxed{b=24}$

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(۹)

$\alpha+\beta=-1$

$\alpha\beta=-1-m^2$

$\alpha^2+\beta^2=s^2-2p=1-2(-m^2-1)=\boxed{3+2m^2}$

$m^2=0 \qquad 3+2m^2 \xrightarrow{m=0} \boxed{3}$

5 | $\frac{-1}{1}$

$y = a(x+1)^2 + 1 \rightarrow 0 = a(x^2+1+2x)+1$ 10

$\alpha + \beta = \frac{-2a}{a} = -2$ $\quad 0 = ax^2 + a + 2ax + 1$

$\alpha\beta = \frac{a+1}{a}$ $\quad \alpha^2 + \beta^2 + 2\alpha\beta = 4$ $\quad 2\alpha\beta = -1$

$\alpha\beta = -\frac{1}{2}$

$\frac{a+1}{a} = -\frac{1}{2}$ $\quad a = -\frac{2}{3}$ $\quad y = -\frac{2}{3}(x+1)^2 + 1$

$x = 0 \quad y = -\frac{2}{3} + 1 \rightarrow \boxed{y = \frac{1}{3}}$