(تکلیف ۲۰) (یازدهم دستی) ۲۰ محیا ...

① $3x^2 - ax + 4 = 0$ $4\beta = \frac{a}{3}$ $\alpha = 3\beta$

اختلاف دو مقدار a در حالت $\alpha = 3\beta$ ؟

$3\beta \times \beta = 3\beta^2$ $3\beta^2 = \frac{4}{3}$ $\beta^2 = \frac{4}{9} = \pm\frac{2}{3}$ ۵

$2 + \frac{2}{3} = \frac{a}{3} \to \frac{8}{3} = \frac{a}{3}$ $\boxed{a_1 = 8}$ پاسخ = $a_1 - a_2 = 16$

$\beta = -\frac{2}{3} \to \alpha = -2$ $-2 - \frac{2}{3} = a$ $\boxed{a_2 = -8}$

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②

$\sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = 5$ ۵

$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = 5$ $\frac{\sqrt{\beta} + \sqrt{\alpha}}{\sqrt{\alpha\beta}} = 5$ $\frac{\sqrt{\alpha} + \sqrt{\beta}}{\frac{1}{6}} = 5 \Rightarrow \sqrt{\alpha} + \sqrt{\beta} = \frac{5}{6}$

$\alpha\beta = \frac{1}{36}$ $\alpha + \beta = \frac{m+14}{36}$ $\alpha + \beta + 2\sqrt{\frac{1}{36}} = \frac{25}{36}$

$mx^2 + 3x + 2 = 0$ $\frac{m+14}{36} = \frac{25}{36} - \frac{12}{36} = \frac{13}{36}$

$-x^2 + 3x + 2 = 0$ $\boxed{\frac{c}{a} = -2}$

$m + 14 = 13$

$\boxed{m = -1}$

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③

$(\alpha + \beta)^2 = 1 \to \alpha^2 + \beta^2 + 2\alpha\beta = 1$ $\left.\begin{array}{l}\alpha^2 + \beta^2 = 5 \\ \alpha + \beta = 1\end{array}\right\}$ $\begin{array}{l}\alpha = 2 \\ \beta = -1\end{array}$

$4 \times (-1)^3 + k + 9 - 2 = 0 \to \boxed{k = -3}$ ✓ ۳

$4 \times 8 + 4k - 18 - 2 = 0 \Rightarrow \boxed{k = -3}$ ✓

Scanned with CamScanner

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$$۳\alpha^۲ + ۲\alpha^۲ = ۲,۵\alpha^۲ + ۲,۵\beta^۲ + ۰,۵\alpha^۲ - ۰,۵\beta^۲$$

$$\frac{۵}{۲}(\alpha^۲+\beta^۲) + \frac{۱}{۲}(\alpha^۲-\beta^۲) \qquad \alpha^۲+\beta^۲ = S^۲ - ۲P \Rightarrow ۳۶ - ۲a$$

$$\alpha^۲-\beta^۲ = (\alpha+\beta)(\alpha-\beta)$$

$$\hookleftarrow -۶(-\sqrt{۳۶-۴a}) = ۶\sqrt{۳۶-۴a}$$

$$\frac{\sqrt{\Delta}}{|a|} = \alpha<\beta \to -\sqrt{\Delta}$$

$$\frac{۵}{۲}(\alpha^۲+\beta^۲) + \frac{۱}{۲}(\alpha^۲-\beta^۲) \Rightarrow ۹۰ - ۵a + ۳\sqrt{۳۶-۴a}$$

$$\hookleftarrow ۱۲\sqrt{۲} + ۸۵$$

$$۹۰ - ۵\alpha = ۸۵ \to \alpha = ۱ \qquad ۳\sqrt{۳۶-۴a} = ۱۲\sqrt{۲} \Rightarrow \boxed{a=۱}$$

۵

$$m^۲ = t \qquad t = \frac{+۷ \overset{قق}{\pm} \sqrt{۶۹}}{۲}$$

$$t^۲ - ۷t - ۵ = ۰$$

$$\Delta = ۴۹ + ۲۰ \qquad m = \pm\sqrt{\frac{۷+\sqrt{۶۹}}{۲}} \qquad S = ۰ \to P = \frac{-۷-\sqrt{۶۹}}{۲}$$

$$۲P^۲ - ۲P\cancel{S} + \cancel{۲S} = ۰$$

$$۲P^۲ = \frac{۴۹+۶۹+۱۴\sqrt{۶۹}}{۴} \times ۲ = \boxed{۵۹+۷\sqrt{۶۹}}$$

۶

$$\alpha+\beta = -\frac{۱}{۲} \qquad \alpha\beta = -\frac{۳}{a}$$

$$\alpha+\beta = -\frac{۱}{۲}$$

$$\alpha+\beta = \frac{a+(-۲)}{۲} \Rightarrow \boxed{a=۱}$$

$$\left(\frac{۱}{۲}+\alpha\right)\left(\frac{۱}{۲}+\beta\right) = \frac{۱}{۴} + (\alpha+\beta)\frac{۱}{۲} + \alpha\beta = \frac{b}{۲}$$

$$\frac{۱}{۴} - \frac{۱}{۴} - \frac{۱۲}{۴} = \frac{b}{۲} \to b = -۶ \qquad \left[\frac{ab}{۴}\right] = \left[\frac{۱\times -۶}{۴}\right] = \left[-\frac{۳}{۲}\right] = -۲$$

(۷)

$S=1 \quad , \quad P=-\frac{b}{a}$

$a\alpha^2-a\alpha-b=0$

$a\beta^2-a\beta-b=0 \rightarrow \beta^2-\beta=\frac{b}{a}$

$40\beta^2+20\alpha^2-20\beta=20\beta^2+20\alpha^2+20\beta^2-20\beta$

$20(S^2-2P)+20(\beta^2-\beta) \Rightarrow 20\left(1+\frac{2b}{a}\right)+20\left(\frac{b}{a}\right)=$

$\hookrightarrow 20+\frac{40b}{a}+20\frac{b}{a}=20+60\frac{b}{a}=17$

$\frac{b}{a}=-\frac{1}{20} \rightarrow b=-\frac{1}{20}a \qquad \frac{\sqrt{\Delta}}{|a|}=\frac{\sqrt{a^2+4ab}}{|a|}$

$\frac{\sqrt{a^2-\frac{a^2}{5}}}{|a|}=\frac{\cancel{|a|}\sqrt{\frac{4}{5}}}{\cancel{|a|}}=\boxed{\frac{2}{\sqrt{5}}}$

پاسخ

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(۸)

$a=2k+1 \times 2k-1=4k^2-1$

$a+1=4k \rightarrow a=4k-1$

$\Big\} \; 4k^2-4k=0$

$4k(k-1)=0 \rightarrow k=1 \quad k\neq 0$

$\boxed{a=3}$

$b=4k'^2+4k'$

ریشه‌های معادله اول $\rightarrow 2k+1 \;,\; 2k-1$

ریشه‌های معادله دوم $\rightarrow 2k' \;,\; 2k'+2$

$3a+1=4k'+2 \qquad \boxed{|a-b|=21}$

$10=4k'+2 \rightarrow k'=2 \qquad \boxed{b=24}$

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(۹)

$\alpha+\beta=-1$

$\alpha\beta=-1-m^2$

$\alpha^2+\beta^2=S^2-2P=1-2(-m^2-1)=\boxed{3+2m^2}$

$m^2=0 \qquad 3+2m^2 \xrightarrow{m=0} \boxed{3}$

5 | -1, 1

$y = a(x+1)^2 + 1 \rightarrow 0 = a(x^2+1+2x)+1$ 10

$\rightarrow 0 = ax^2 + a + 2ax + 1$ 5

$\alpha + \beta = \frac{-2a}{a} = -2$

$\alpha\beta = \frac{a+1}{a}$ $\quad \alpha^2 + \beta^2 + 2\alpha\beta = 4$ $\quad 2\alpha\beta = -1$

$\alpha\beta = -\frac{1}{2}$

$\frac{a+1}{a} = -\frac{1}{2}$ $\quad a = -\frac{2}{3}$ $\quad y = -\frac{2}{3}(x+1)^2 + 1$

$x = 0$ $\quad y = -\frac{2}{3} + 1 \rightarrow \boxed{y = \frac{1}{3}}$