

سوال (۱)

$$3x^2 - ax + 4 = 0$$

$$x, 3x \rightarrow P = \frac{4}{3} = 3x^2 \rightarrow x = \pm \frac{2}{3}$$

$$\rightarrow S = \frac{a}{3} = 4x \rightarrow x = \frac{a}{12} \rightarrow \begin{cases} \frac{a}{12} = \frac{2}{3} \rightarrow a = 8 \\ \frac{a}{12} = -\frac{2}{3} \rightarrow a = -8 \end{cases}$$

اختلاف مقادیر = **(۱۴)**

سوال (۲)

$$34x^2 - (m+14)x + 1 = 0$$

$$mx^2 + 3x + 2 = 0$$

ریشه‌های عادی اول: x, y

$$\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} = 2 \Rightarrow \frac{\sqrt{y} + \sqrt{x}}{\sqrt{xy}} = 2, \quad x+y = \frac{m+14}{34}, \quad xy = \frac{1}{34}$$

$$\Rightarrow \frac{x+y+2\sqrt{xy}}{xy} = 2 \Rightarrow \frac{m+14}{34} + 2\sqrt{\frac{1}{34}} = 2 \cdot \frac{1}{34} \Rightarrow \frac{m+14}{34} + \frac{2}{\sqrt{34}} = \frac{2}{34} \Rightarrow m+14+12 = 0 \Rightarrow m = -1$$

$$mx^2 + 3x + 2 = 0 \xrightarrow{m=-1} -x^2 + 3x + 2 = 0 \rightarrow \frac{2}{-1} = \boxed{-2}$$

سوال (۳)

$$x^2 + kx^2 - 9x - 2 = 0, \quad \alpha + \beta = 1, \quad \alpha\beta = -2 \quad k = ?$$

$$\alpha\beta = -2 \rightarrow \alpha = -\frac{2}{\beta}$$

$$\alpha + \beta = 1 \xrightarrow{\alpha = -\frac{2}{\beta}} -\frac{2}{\beta} + \beta = 1 \rightarrow -2 + \beta^2 = \beta \rightarrow \beta^2 - \beta - 2 = 0 \rightarrow (\beta - 2)(\beta + 1) = 0 \Rightarrow \begin{cases} \beta = 2 \\ \beta = -1 \end{cases}$$

$$\begin{aligned} x^2 + kx^2 - 9x - 2 = 0 \quad x=2 &\rightarrow 4 + 4k - 18 - 2 = 0 \rightarrow k = -3 \\ x^2 + kx^2 - 9x - 2 = 0 \quad x=-1 &\rightarrow 1 - k - 9 - 2 = 0 \rightarrow k = -3 \end{aligned}$$

$$\boxed{k = -3}$$

سوال (۴)

$$x^2 + 4x + a = 0 \quad \alpha < \beta < 0, \quad 2\alpha^2 + 2\beta^2 = 12\sqrt{17} + 18 \quad a = ?$$

$$2\alpha^2 + 2\beta^2 = 2\alpha(\alpha + \beta) + 2\alpha(\alpha - \beta) = \frac{a}{2}(s^2 - 4p) + \frac{1}{2}(\beta - \alpha) \cdot \frac{\sqrt{\Delta}}{a}$$

$$\begin{aligned} x^2 + 4x + a &\xrightarrow{\frac{b}{a}} s = -4 \\ &\xrightarrow{\frac{-b \pm \sqrt{\Delta}}{a}} p = a \\ &\xrightarrow{\frac{\sqrt{\Delta}}{a}} \sqrt{\Delta} = \sqrt{16 - 4a} \end{aligned} \Rightarrow \frac{a}{2}(16 - 4a) + \frac{1}{2}(\sqrt{16 - 4a}) = 9\sqrt{17} + 18 \Rightarrow a = 1$$

سوال (۵)

$$x^2 - 7x^2 - 5 = 0 \quad P: \text{حاصلضرب ریشه‌ها} \quad S: \text{مجموع ریشه‌ها} \quad 2p^2 - 3sp + 5s = ?$$

$$\rightarrow y = x^2 \Rightarrow y^2 - 7y - 5 = 0 \xrightarrow{\text{ریشه‌ها}} y = \frac{7 \pm \sqrt{49 + 20}}{2} \Rightarrow x = \pm \sqrt{y} \rightarrow x = \pm \sqrt{\frac{7 \pm \sqrt{69}}{2}}$$

$$\begin{aligned} S &= \sqrt{\frac{7+\sqrt{69}}{2}} + (-\sqrt{\frac{7+\sqrt{69}}{2}}) = 0 \\ P &= \left(\sqrt{\frac{7+\sqrt{69}}{2}}\right)\left(-\sqrt{\frac{7+\sqrt{69}}{2}}\right) = -\left(\frac{7+\sqrt{69}}{2}\right) \end{aligned}$$

$$\Rightarrow 2p^2 - 3sp + 5s = 2\left(-\frac{7+\sqrt{69}}{2}\right)^2 - 3(0)\left(-\frac{7+\sqrt{69}}{2}\right) + 5(0) = \frac{(7+\sqrt{69})^2}{2} = \frac{49 + 7\sqrt{69} + 69}{2} = \frac{118 + 7\sqrt{69}}{2}$$

سوال (۶)

$$2x^2 - ax + b = 0$$

$$2ax^2 + ax - 4 = 0$$

$$\left[\frac{ab}{f}\right] = ?$$

$$\left[\frac{ab}{f}\right] = \boxed{-2}$$

$$2ax^2 + ax - 4 = 0 \xrightarrow{\text{ریشه‌های معلوم } x, y} x+y = -\frac{1}{2}, \quad xy = -\frac{2}{a}$$

$$2x^2 - ax + b = 0 \xrightarrow{\text{ریشه‌های معلوم}} x + \frac{1}{x}, \quad y + \frac{1}{y} \rightarrow (x + \frac{1}{x}) + (y + \frac{1}{y}) = \frac{a}{2} \rightarrow x+y + \frac{1}{x} + \frac{1}{y} = \frac{a}{2}$$

$$-\frac{1}{2} + 1 = \frac{a}{2} \rightarrow \frac{1}{2} = \frac{a}{2} \rightarrow a = 1, \quad (x + \frac{1}{x})(y + \frac{1}{y}) = xy + \frac{x}{y} + \frac{y}{x} + \frac{1}{xy} = -\frac{2}{a} + \frac{-\frac{1}{2}}{\frac{1}{2}} + \frac{1}{\frac{1}{2}} = -\frac{2}{a}$$

$$\left[\frac{ab}{f}\right] = \left[\frac{1 \cdot (-4)}{2}\right] = \left[-\frac{4}{2}\right] = \left[-\frac{2}{1}\right] = \boxed{-2}$$

سوال ۷

$$ax^2 - ax - b = 0, \quad 4 \cdot \beta^2 + 2 \cdot a^2 - 2 \cdot \beta = 17 \Rightarrow 4 \cdot \beta^2 + 2 \cdot (1 - \beta)^2 - 2 \cdot \beta = 17$$

$$S=1 \Rightarrow \alpha + \beta = 1 \Rightarrow a = 1 - \beta$$

$$\Rightarrow 4 \cdot \beta^2 + 2 \cdot 1 - 4 \cdot \beta = 17$$

$$\Rightarrow 4 \cdot \beta^2 - 4 \cdot \beta + 2 = 17$$

$$\Rightarrow 4 \cdot \beta^2 - 4 \cdot \beta + 1 = 0 \xrightarrow{ax^2 - ax - b} a = -2 \cdot b$$

$$\Rightarrow -2 \cdot b n^2 + 2 \cdot b n - b = 0 \rightarrow \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{4 \cdot b^2 - 4 \cdot b^2}}{1 - 2 \cdot b} = \frac{1 \cdot |b| \sqrt{0}}{1 - 2 \cdot b} = \frac{1 \cdot \sqrt{0} \cdot |b|}{2 \cdot |b|} = \frac{2}{\sqrt{0}}$$

سوال ۸

$$x^2 - (a+1)x + a = 0 \quad x^2 - (3a+1)x + b = 0$$

$$\xrightarrow{\text{ریشه ها}} x, x+2 \rightarrow (x)(x+2) = a \rightarrow x^2 + 2x = a$$

$$x + (x+2) = a+1 \rightarrow 2x+2 = a+1$$

$$2x+2 = a+1 \xrightarrow{x=1} 2+2 = a+1 \rightarrow a=3$$

$$\left\{ \begin{array}{l} x=1 \\ x=3 \end{array} \right\} \text{ ریشه های معادله}$$

$$x^2 - (3a+1)x + b = 0$$

$$\xrightarrow{\text{ریشه ها}} y, y+2 \quad (y)(y+2) = b$$

$$y + (y+2) = 3a+1 \rightarrow 2y+2 = 3a+1 \xrightarrow{a=3} 2y+2 = 10 \rightarrow 2y = 8 \rightarrow y = 4 \Rightarrow \left\{ \begin{array}{l} y=4 \\ y=6 \end{array} \right\} \text{ ریشه های معادله}$$

$$\text{اختلاف حاصل ضرب ریشه های دو معادله} = (4 \times 6) - (3 \times 1) = 24 - 3 = 21$$

سوال ۹

$$x^2 + x - 1 - m^2 = 0 \quad a^2 + \beta^2 = ?$$

$$a^2 + \beta^2 = (a+\beta)^2 - 2a\beta = (-1)^2 - 2(-1-m^2) = 1 + 2(1+m^2) = 1 + 2 + 2m^2 = 3 + 2m^2$$

$$a+\beta = -1$$

$$a\beta = -(1+m^2)$$

$$3 + 2m^2 \xrightarrow{\text{کمترین مقدار عبارت فوق}} m=0 \rightarrow 3 + 2(0) = 3$$

سوال ۱۰

$$A = (3, y) \quad B = (-5, y) \quad \text{عرض راس محسوس} = 1 \quad a^2 + \beta^2 = 5$$

$$\text{طول عرضی محسوس} = \frac{3 + (-5)}{2} = -1 \Rightarrow \text{رأی} = \frac{-1}{+1} \rightarrow a(x-h)^2 + k \xrightarrow{\frac{h=-1}{k=1}} a(x+1)^2 + 1 = ax^2 + 2ax + a + 1$$

$$a^2 + \beta^2 = 0 \rightarrow 5^2 - 2p = 0 \rightarrow 25 - \frac{2a \cdot 2}{a} = 0 \rightarrow \frac{25 - 4a}{a} = 0 \rightarrow 25 - 4a = 0 \rightarrow 4a = 25 \rightarrow a = \frac{25}{4}$$

$$\Rightarrow y = \frac{25}{4} x^2 + \frac{25}{2} x + \frac{25}{4} \Rightarrow \text{نقطه از مبدأ} = c \rightarrow \frac{25}{4}$$