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سوال ۱

$$3x^2 - ax + 4 = 0$$

$$x, 3x \rightarrow P = \frac{4}{3} = 3x^2 \rightarrow x = \pm \frac{2}{3}$$

$$\rightarrow S = \frac{a}{3} = 4x \rightarrow x = \frac{a}{12} \rightarrow \begin{cases} \frac{a}{12} = \frac{2}{3} \rightarrow a = 8 \\ \frac{a}{12} = -\frac{2}{3} \rightarrow a = -8 \end{cases}$$

اختلاف مقادیر = **(14)**

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سوال ۲

$$34x^2 - (m+14)x + 1 = 0$$

$$mx^2 + 3x + 2 = 0$$

ریشه‌های عادی اول: x, y

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$$\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} = 2 \Rightarrow \frac{\sqrt{y} + \sqrt{x}}{\sqrt{xy}} = 2, x+y = \frac{m+14}{34}, xy = \frac{1}{34}$$

$$\Rightarrow \frac{x+y+2\sqrt{xy}}{xy} = 2 \Rightarrow \frac{m+14}{34} + 2\sqrt{\frac{1}{34}} = 2 \cdot \frac{1}{34} \Rightarrow \frac{m+14}{34} + \frac{2}{\sqrt{34}} = \frac{2}{34} \Rightarrow m+14+12 = 0 \rightarrow m = -1$$

$$mn^2 + 3n + 2 = 0 \xrightarrow{m=-1} -n^2 + 3n + 2 = 0 \rightarrow \frac{2}{-1} = \boxed{-2}$$

$$3x^2 + kx^2 - 9x - 2 = 0, a+\beta = 1, a\beta = -2, k = ?$$

$$a\beta = -2 \rightarrow a = -\frac{2}{\beta}$$

$$a+\beta = 1 \xrightarrow{a=-\frac{2}{\beta}} -\frac{2}{\beta} + \beta = 1 \rightarrow -2 + \beta^2 = \beta \rightarrow \beta^2 - \beta - 2 = 0 \rightarrow (\beta-2)(\beta+1) = 0 \Rightarrow \begin{cases} \beta = 2 \\ \beta = -1 \end{cases}$$

$$3x^2 + kx^2 - 9x - 2 = 0 \xrightarrow{x=2} 12 + k(4) - 18 - 2 = 0 \rightarrow k = -3$$

$$\boxed{k = -3}$$

سوال ۳

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$$x^2 + 4x + a = 0, a < \beta < 0, 2a^2 + 2\beta^2 = 12\sqrt{2} + 18, a = ?$$

$$2a^2 + 2\beta^2 = 2(a^2 + \beta^2) = 2(a^2 + (a+\beta)^2 - 2a\beta) = \frac{2}{2}(s^2 - 2p) + \frac{1}{2}(\beta^2 - \frac{\sqrt{\Delta}}{a})$$

$$x^2 + 4x + a = 0 \xrightarrow{\frac{b}{a}} s = -4, p = a, \frac{\sqrt{\Delta}}{a} = \sqrt{16-4a} \Rightarrow \frac{2}{2}(16-4a) + \frac{1}{2}(16-4a) = 9 \cdot 2 + 3\sqrt{16-4a} = 12\sqrt{2} + 18 \Rightarrow 16-4a = 18 \Rightarrow a = 1$$

$$x^2 - 7x^2 - 5 = 0, P: \text{حاصلضرب ریشه‌ها}, S: \text{مجموع ریشه‌ها}, 2p^2 - 3sp + 5s = ?$$

سوال ۵

$$\rightarrow y = x^2 \Rightarrow y^2 - 7y - 5 = 0 \xrightarrow{\text{ریشه‌ها}} y = \frac{7 \pm \sqrt{49+20}}{2} \Rightarrow x = \pm \sqrt{y} \rightarrow x = \pm \sqrt{\frac{7 \pm \sqrt{69}}{2}}$$

$$\Rightarrow S = \sqrt{\frac{7+\sqrt{69}}{2}} + (-\sqrt{\frac{7+\sqrt{69}}{2}}) = 0$$

$$P = (\sqrt{\frac{7+\sqrt{69}}{2}})(-\sqrt{\frac{7+\sqrt{69}}{2}}) = -(\frac{7+\sqrt{69}}{2})$$

$$\Rightarrow 2p^2 - 3sp + 5s = 2(-\frac{7+\sqrt{69}}{2})^2 - 3(0)(-\frac{7+\sqrt{69}}{2}) + 5(0) = 2(\frac{(7+\sqrt{69})^2}{4}) = \frac{(7+\sqrt{69})^2}{2} = 59 + 7\sqrt{69}$$

$$2x^2 - ax + b = 0, 2ax^2 + ax - 4 = 0, [\frac{ab}{f}] = ?$$

سوال ۴

$$2ax^2 + ax - 4 = 0 \xrightarrow{\text{ریشه‌های عادی}} x+y = -\frac{1}{2}, xy = -\frac{4}{a}$$

$$2x^2 - ax + b = 0 \xrightarrow{\text{ریشه‌های عادی}} x+\frac{1}{y}, y+\frac{1}{y} \rightarrow (x+\frac{1}{y}) + (y+\frac{1}{y}) = \frac{a}{2} \rightarrow x+y+\frac{1}{y} = \frac{a}{2} \rightarrow$$

$$-\frac{1}{y} + \frac{1}{y} = \frac{a}{2} \rightarrow \frac{1}{y} = \frac{a}{2} \rightarrow a=1, (x+\frac{1}{y})(y+\frac{1}{y}) = xy + \frac{x}{y} + \frac{y}{y} + \frac{1}{y^2} = -\frac{4}{a} + \frac{1}{y} + \frac{1}{y} = -\frac{4}{a}$$

$$[\frac{ab}{f}] = [\frac{1 \cdot (-4)}{2}] = [-\frac{4}{2}] = [-\frac{2}{1}] = \boxed{-2}$$

سوال ۷

$$ax^2 - ax - b = 0, \quad 4 \cdot \beta^2 + 2 \cdot a^2 - 2 \cdot \beta = 17 \Rightarrow 4 \cdot \beta^2 + 2 \cdot (1 - \beta)^2 - 2 \cdot \beta = 17$$

$$S=1 \Rightarrow \alpha + \beta = 1 \Rightarrow a = 1 - \beta$$

$$\Rightarrow 4 \cdot \beta^2 + 2 \cdot 1 - 4 \cdot \beta = 17$$

$$\Rightarrow 4 \cdot \beta^2 - 4 \cdot \beta + 2 = 17$$

$$\Rightarrow 4 \cdot \beta^2 - 4 \cdot \beta + 1 = 0 \xrightarrow{ax^2 - ax - b} a = -2 \cdot b$$

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$$\Rightarrow -2 \cdot b n^2 + 2 \cdot b n - b = 0 \rightarrow \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{4 \cdot b^2 - 4 \cdot b^2}}{1 - 2 \cdot b} = \frac{1 \cdot |b| \sqrt{0}}{1 - 2 \cdot b} = \frac{1 \cdot \sqrt{0} \cdot |b|}{2 \cdot |b|} = \frac{2}{\sqrt{0}}$$

سوال ۸

$$x^2 - (a+1)x + a = 0 \quad x^2 - (3a+1)x + b = 0$$

$$\left[\begin{array}{l} x, x+2 \rightarrow (x)(x+2) = a \rightarrow x^2 + 2x = a \\ x + (x+2) = a+1 \rightarrow 2x+2 = a+1 \end{array} \right] \rightarrow 2x+2 = (x^2 + 2x) + 1 \rightarrow 1 = x^2 \rightarrow x = \pm 1$$

$$2x+2 = a+1 \xrightarrow{x=1} 2+2 = a+1 \rightarrow a=3$$

$$\{1, 3\} \leftarrow \text{ریشه های معادله}$$

منفی یک قطبی نیست و غیر قابل قبول است

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$$x^2 - (3a+1)x + b = 0$$

$$y, y+2 \rightarrow (y)(y+2) = b$$

$$(y)(y+2) = b$$

$$y + (y+2) = 3a+1 \rightarrow 2y+2 = 3a+1 \xrightarrow{a=3} 2y+2 = 10 \rightarrow 2y = 8 \rightarrow y = 4 \Rightarrow \{2, 4\}$$

$$\text{اختلاف حاصل ضرب ریشه های دو معادله} = (4 \times 4) - (3 \times 1) = 16 - 3 = 13$$

سوال ۹

$$x^2 + x - 1 - m^2 = 0$$

$$a^2 + \beta^2 = ?$$

$$a^2 + \beta^2 = (a + \beta)^2 - 2a\beta = (-1)^2 - 2(-1 - m^2) = 1 + 2(1 + m^2) = 1 + 2 + 2m^2 = 3 + 2m^2$$

$$\left[\begin{array}{l} a + \beta = -1 \\ a\beta = -(1 + m^2) \end{array} \right]$$

$$3 + 2m^2 \rightarrow \text{کمترین مقدار عبارت وقتی } m=0 \rightarrow 3 + 2(0) = 3$$

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$$A = (3, y) \quad B = (-5, y) \quad \text{عرض راس محسوس} = 1 \quad a^2 + \beta^2 = 5$$

سوال ۱۰

$$\text{طول عرضی} = \frac{3 + (-5)}{2} = -1 \Rightarrow \text{رأی} = \frac{-1}{+1} \rightarrow a(x-h)^2 + k \xrightarrow[h=1]{k=1} a(x+1)^2 + 1 = ax^2 + 2ax + a + 1$$

$$\left[\begin{array}{l} p = \frac{a-1}{a} \\ S = 2 \end{array} \right]$$

$$a^2 + \beta^2 = 0 \rightarrow 5 - 2p = 0 \rightarrow 5 - \frac{2(a-1)}{a} = 0 \rightarrow \frac{5a - 2a + 2}{a} = 0 \rightarrow 3a = -2 \rightarrow a = -\frac{2}{3}$$

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$$\Rightarrow y = -\frac{2}{3}x^2 + \frac{4}{3}x + \frac{1}{3} \Rightarrow \text{نقطه اوج} = \left(\frac{-1}{3} \right)$$