نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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$\alpha = 2\beta$ و $\alpha\beta = \frac{9}{2}$

$\alpha = 2 \cdot \frac{3}{2} = 3 \Rightarrow \alpha = 3$

$2\beta \times \beta = 2\beta^2 = \frac{9}{2}$ $\beta^2 = \frac{9}{4} \rightarrow \beta = \pm\frac{3}{2}$

$3 + \frac{3}{2} = \frac{9}{2} \rightarrow \frac{9}{2} = \frac{9}{2} \Rightarrow a_1 = 9$

$\beta = -\frac{3}{2} \rightarrow \alpha = -3$ $-3 - \frac{3}{2} = \frac{9}{2}$ $a_2 = -9$

$9 - (-9) = 18$

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$\sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = 2 \rightarrow \frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \frac{\sqrt{\beta} + \sqrt{\alpha}}{\sqrt{\alpha\beta}} = 2$

$\frac{\sqrt{\alpha} + \sqrt{\beta}}{\frac{1}{3}} = 2 \rightarrow \sqrt{\alpha} + \sqrt{\beta} = \frac{2}{3}$ $\alpha\beta = \frac{1}{36}$ $\alpha + \beta = \frac{m+14}{36}$

$\alpha + \beta + 2\sqrt{\frac{1}{36}} = \frac{12}{36} \rightarrow \frac{m+14}{36} = \frac{24 - 12}{36} = \frac{12}{36} \rightarrow m + 14 = 24$

$m = 10$

$mn^2 - 3n + 2 = 0$

$-n^2 + 3n + 2 = 0$

$P = \frac{c}{a} = -2$

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$(\alpha + \beta)^2 = 1 \rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 1$ ($2\alpha\beta = -4$)

$\alpha^2 + \beta^2 = 5$

$\alpha^2 + \beta = 1 \rightarrow \alpha = 2$, $\beta = 1$

$4(-1)^2 + k + 9 - 2 = 0 \rightarrow k = -3$

$\rightarrow k = -3$

$4 \times 8 + 4k - 18 - 2 = 0 \rightarrow k = -3$

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$2\alpha^2 + 2\beta^2 = 2, 5\alpha^2 + 3, 5\beta^2 = 0, 5\alpha^2 = 0, 5\beta^2$

$\frac{5}{2}(\alpha^2 + \beta^2) + \frac{1}{2}(\alpha^2 - \beta^2)$

$\alpha^2 + \beta^2 = 49 - 24$

$\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta) = -9\left(-\sqrt{49 - 24}\right) = 9\sqrt{49 - 24}$

$\frac{5}{2}(\alpha^2 + \beta^2) + \frac{1}{2}(\alpha^2 - \beta^2) \rightarrow 90 - 45 + 3\sqrt{49 - 24} = 12\sqrt{2} + 45$

$\alpha < \beta \rightarrow -\sqrt{\Delta}$

$3\sqrt{49 - 24} = 12\sqrt{2} \Rightarrow a = 1$

$90 - 45\alpha = 45 \rightarrow \alpha = 1$

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$n^2 = t \rightarrow t^2 - 7t - 5 = 0$

$t = \frac{7 \pm \sqrt{49}}{2} \rightarrow n = \pm\sqrt{\frac{7 + \sqrt{49}}{2}}$

$S = 0 \Rightarrow P = -\frac{7 - \sqrt{49}}{2}$ $2P^2 - 4PS + 2S^2 \rightarrow 2P^2 = 2 \times \frac{54 + 49 + 14\sqrt{49}}{4} = 59 + 7\sqrt{49}$

$\alpha+\beta=-\frac{1}{2}$ ، $\alpha\beta=-\frac{2}{a}$

$\alpha+\frac{1}{\beta}+\beta+\frac{1}{\alpha}=\frac{a}{2}$ ، $\alpha+\beta+1=\frac{\alpha}{2}$

$\alpha+\beta=-\frac{1}{2}$

$\alpha+\beta=\frac{a-2}{2}\rightarrow a=1$ $\quad \left(\frac{1}{\alpha}+\alpha\right)\left(\frac{1}{\beta}+\beta\right)=\frac{1}{\alpha\beta}+(\alpha+\beta)\frac{1}{\alpha\beta}+\alpha\beta=\frac{b}{2}$

$\frac{1}{4}-\frac{1}{2}-\frac{12}{4}=\frac{b}{2}\rightarrow b=-6$

$\left[\frac{ab}{6}\right]=\left[\frac{1(-6)}{4}\right]=\left[-\frac{3}{2}\right]=-2$

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$a\alpha^2-a\alpha-b=0$ $\quad S=1$

$a\beta^2-\alpha\beta-b=0\rightarrow\beta^2-\beta=\frac{b}{a}$ $\quad P=-\frac{b}{a}$

$4\beta^2+20\alpha^2-20\beta=20\beta^2+20\alpha^2+20\beta^2-20\beta$

$20(S^2-2P)+20(\beta^2-\beta)\rightarrow 20\left(1+\frac{2b}{a}\right)+20\left(\frac{b}{a}\right)=20+40\frac{b}{a}+20\frac{b}{a}=17$

$\frac{b}{a}=-\frac{1}{20}\rightarrow b=-\frac{1}{20}a\rightarrow\frac{\sqrt{\Delta}}{|a|}=\frac{\sqrt{a^2+4ab}}{|a|}\rightarrow\frac{|a|\sqrt{\frac{4}{5}}}{|a|}=\sqrt{\frac{4}{5}}=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$

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$a=(2k+1)^2-(2k-1)^2\rightarrow 4k^2$

$a+1=4k\rightarrow a=4k-1$

$\left.\begin{array}{l}\\ \\ \end{array}\right\}\rightarrow 4k^2-4k=0$

$4k(k-1)=0$

$k=1,0$

$\rightarrow a=3$

$b=4k^2+4k$

$2a+1=4k+2$

$10=4k+2\rightarrow k^2=2\rightarrow b=24$

$|a-b|=|3-24|=21$

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$\alpha+\beta=1$ ، $\alpha\beta=-1-m^2$ $\quad \alpha^2+\beta^2=S^2-2P$

$1-2(-m^2-1)=4+2m^2$

$m^2=0\rightarrow 4+2m^2=4$

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$y=a(x+1)^2+1\rightarrow 0=a(x^2+2x)+1\rightarrow 0=ax^2+2ax+1$

$\alpha+\beta=\frac{-2a}{a}=-2$ $\quad \alpha\beta=\frac{a+1}{a}$ $\quad \alpha^2+\beta^2+2\alpha\beta=4\rightarrow\alpha\beta=-\frac{1}{2}$

$\frac{a+1}{a}=-\frac{1}{2}\rightarrow a=-\frac{2}{3}$ $\quad y=-\frac{2}{3}(x+1)^2+1$

$x=0\rightarrow y=-\frac{2}{3}+1=\frac{1}{3}$

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