ش ۱۹ سنجش صفر یازدهم [illegible]

س ۱

$3x^2-ax+2=0 \qquad x,3x \to P=\frac{2}{3}=3x^2 \to x=\pm\frac{2}{3}$

$\hookrightarrow S=\frac{a}{3}=4x \to x=\frac{a}{12}$

$\Rightarrow \begin{cases} \frac{a}{12}=\frac{-2}{3} \to a=-8 \\ \frac{a}{12}=\frac{2}{3} \to a=8 \end{cases} \longrightarrow \boxed{16}$

س ۲

$\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{y}}=5 \Rightarrow \frac{\sqrt{y}+\sqrt{x}}{\sqrt{xy}}=5 \,,\; x+y=\frac{m+14}{36} \,,\; xy=\frac{1}{36}$

$\Longrightarrow \frac{x+y+2\sqrt{xy}}{xy}=25 \to \frac{m+14}{36}+2\sqrt{\frac{1}{36}}=25\left(\frac{1}{36}\right) \to \frac{m+14}{36}+\frac{2}{6}=\frac{25}{36} \to m+14+12=25 \to m=-1$

$mx^2+3x+2=0 \xrightarrow{m=-1} -x^2+3x+2=0 \longrightarrow \frac{2}{-1}=\boxed{-2}$

س ۳

$\alpha\beta=-2 \to \alpha=\frac{-2}{\beta}$

$\alpha+\beta=1 \xrightarrow{\alpha=\frac{-2}{\beta}} \frac{-2}{\beta}+\beta=1 \to -2+\beta^2=\beta \to \beta^2-\beta-2=0 \to (\beta-2)(\beta+1)=0 \Rightarrow \begin{cases} \beta=2 \\ \beta=-1 \end{cases}$

$4x^3+kx^2-9x-2=0 \xrightarrow{x=2} 4(2)^3+k(2)^2-9(2)-2=0 \to k=-3$

$4x^3+kx^2-9x-2=0 \xrightarrow{x=-1} 4(-1)^3+k(-1)^2-9(-1)-2=0 \to k=-3$

$\boxed{k=-3}$

سوال ۴

$3\alpha^2+2\beta^2=2.5(\alpha^2+\beta^2)+0.5(\underbrace{\alpha^2-\beta^2}_{(\alpha-\beta)(\alpha+\beta)})=\frac{5}{2}(S^2-2P)+$

$\frac{1}{2}\left(|S|\frac{\sqrt{\Delta}}{a}\right)$

$x^2+6x+a \xrightarrow{\frac{-b}{a}} S=-6$

$\xrightarrow{\frac{c}{a}} P=a$

$\to \frac{\sqrt{\Delta}}{a}=\sqrt{36-4a}$

$\Rightarrow \frac{5}{2}(36-2a)+(3\times\sqrt{36-4a})=$

$90-5a+3\sqrt{36-4a}=12\sqrt{2}+85$

$3\sqrt{32}=12\sqrt{2} \Rightarrow 5a=5$

$\to \boxed{a=1}$

سوال ۵

$x^4-7x^2-5=0 \to y=x^2 \to y^2-7y-5=0 \xrightarrow{\text{ریشه‌ها}} y=\frac{7\pm\sqrt{69}}{2}$

$\Rightarrow x=\pm\sqrt{y} \to x=\pm\sqrt{\frac{7+\sqrt{69}}{2}}$

مجموع ریشه‌ها: $S=\sqrt{\frac{7+\sqrt{69}}{2}}+\left(-\sqrt{\frac{7+\sqrt{69}}{2}}\right)=0$

حاصلضرب ریشه‌ها $P=\left(\sqrt{\frac{7+\sqrt{69}}{2}}\right)\left(-\sqrt{\frac{7+\sqrt{69}}{2}}\right)=-\left(\frac{7-\sqrt{69}}{2}\right)$

$\to 2P^2-3SP+2S=2\left(-\left(\frac{7+\sqrt{69}}{2}\right)\right)^2-3(0)\left(\frac{-7+\sqrt{69}}{2}\right)+2(0)=$

$2\left(\frac{(7+\sqrt{69})^2}{4}\right)=\boxed{\frac{(7+\sqrt{69})^2}{2}=59+7\sqrt{69}}$

سوال ۶

$2ax^2+ax-6=0 \xrightarrow[x,y]{\text{ریشه‌های معادله}} x+y=\frac{-1}{2},\ xy=\frac{-3}{a}$

$2x^2-ax+b=0 \xrightarrow{\text{ریشه‌های معادله}} x+\frac{1}{2},\ y+\frac{1}{2} \to (x+\frac{1}{2})+(y+\frac{1}{2})=\frac{a}{2} \to$

$\underbrace{x+y}_{\frac{-1}{2}}+1=\frac{a}{2} \to \frac{-1}{2}+1=\frac{a}{2} \to \frac{1}{2}=\frac{a}{2} \to a=1,\ (x+\frac{1}{2})(y+\frac{1}{2})=xy+\frac{x}{2}+\frac{y}{2}+\frac{1}{4}$

$=\frac{-3}{a}+\frac{\frac{-1}{2}}{2}+\frac{1}{4}=\frac{-3}{a}=\frac{b}{2} \xrightarrow{a=1} -3=\frac{b}{2} \to b=-6$

$\boxed{\left[\frac{ab}{4}\right]=\left[\frac{1\times(-6)}{4}\right]=\left[\frac{-6}{4}\right]=\left[\frac{-3}{2}\right]=-2}$

س۷:

$\alpha+\beta = \frac{-b}{a} = \frac{a}{a} = 1 \to \beta = 1-\alpha$ $\quad 40(1-\alpha)^2 + 20\alpha^2 - 20(1-\alpha) = 17$

$60\alpha^2 - 60\alpha + 20 = 17$

$|\alpha-\beta| = \sqrt{S^2-4P} = \sqrt{1-\frac{4}{20}} = \boxed{\frac{2}{\sqrt{5}}}$ $\quad 20\alpha^2 - 20\alpha + 1 = 0 \to S=1, P=\frac{1}{20}$

س۸:

① $x^2-(a+1)x+a=0$

↳ $x, x+2$ $\quad S = x+x+2 = 2x+2 = a+1 \to a = 2x+1$

$\boxed{\text{۳ و ۱، پس ریشه‌ها}} \to a=3$

$P = x(x+2) = a \xrightarrow{a=2x+1} x^2+2x = 2x+1$

$x^2 = 1 \to x = \pm 1$

② $\to 2k, 2k+2$

$S = 4k+2 = 3a+1 \xrightarrow{a=3} 4k+2 = 10 \to k=2$

ریشه‌ها: $4, 6 \to b = 24$

اختلاف حاصل ضرب $= |(3\times1)-(6\times4)| = \boxed{21}$

س۹:

$x^2+x-(1+m^2)=0$

$a+b=-1 \quad , ab = -(1+m^2)$

$a^2+b^2 = (a+b)^2-2ab \to (-1)^2-2(-(1+m^2)) = \underline{3+2m^2}$ ← کمترین مقدار زمانی است که $m^2=0$ باشد پس:

$3+2m^2 \xrightarrow{m=0} 3+2(0)^2 = \boxed{3}$

س۱۰:

$\underline{A(3,y), B(-5,y)}$

↳ $x_S = \frac{3-5}{2} = -1$

$y = a(x-h)^2+k \Rightarrow y = a(x+1)^2+1 \to y = ax^2+2ax+a+1 \quad S=-2, P=\frac{a+1}{a}$

$\alpha^2+\beta^2 = 5 \to (\alpha+\beta)^2 - 2\alpha\beta = 5 \to 4 - 2\left(\frac{a+1}{a}\right) = 5$

$\frac{a+1}{a} = \frac{-1}{2} \to 2a+2 = -a$ $\quad$ عرض از مبدا $\to a+1 = -\frac{2}{3}+1 = \boxed{\frac{1}{3}}$

$3a = -2$

$a = -\frac{2}{3}$