۱-

$\alpha = 3\beta$

$\frac{c}{a} = \alpha \cdot \beta \Rightarrow \frac{4}{3} = 3\beta^2 \to 9\beta^2 = 4 \xrightarrow{\text{جذر}} \beta = \pm\frac{2}{3}$ و $\alpha = \pm 2$

$\beta^2 = \frac{4}{9}$

$\frac{a}{3} = \frac{2}{3} + 2 = \frac{8}{3}$ یا $-\frac{2}{3} - 2 = -\frac{8}{3}$

$a = \pm 8$ یا

اختلاف دو مقدار $a = 8 + 8 = 16$

۲-

$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = 5 \xrightarrow[\text{دو طرف}]{\text{توان ۲}} \frac{\beta + \alpha + 2\sqrt{\alpha \cdot \beta}}{\alpha \cdot \beta} = 25 \Rightarrow \frac{m+14+12}{36} = \frac{25}{36} \to m = -1$

($\beta + \alpha = \frac{m+14}{36}$ ، $\sqrt{\alpha\cdot\beta} = \frac{1}{6}$ ، $\alpha \cdot \beta = \frac{1}{36}$)

$-21x^2 + 3x + 2 = 0 \to P = -2$

۳-

$\begin{cases} \alpha + \beta = 1 \\ \alpha \cdot \beta = -2 \end{cases} \Rightarrow (1-\beta)\beta = -2$

$\beta^2 - \beta - 2 = 0 \to \beta = 2$ و $\alpha = -1 \xrightarrow{\beta = 2} 32 + 4K - 18 - 2 = 0 \to K = -3$

$\to \beta = -1$ و $\alpha = 2$

$\xrightarrow{\beta = -1} -4 + K + 9 - 2 = 0 \to K = -3$

۴-

$\Delta = 36 - 4a \Rightarrow x = \frac{-6 \pm \sqrt{36 - 4a}}{2} = -3 + \sqrt{9-a} = \beta$ ، $-3 - \sqrt{9-a} = \alpha$

$3(-3 - \sqrt{9-a})^2 + 2(-3 + \sqrt{9-a})^2 = 12\sqrt{2} + 85 \Rightarrow 3(18 - a + 6\sqrt{9-a}) + 2(18 - a - 6\sqrt{9-a}) = 12\sqrt{2} + 85$

$\xrightarrow[\text{عبارت}]{\text{قسمت مهم}} 18\sqrt{9-a} - 12\sqrt{9-a} = 12\sqrt{2} \to \sqrt{9-a} = \sqrt{8} \to 9 - a = 8$

$a = 1$

۵-

$x^2 = z \to z^2 - 7z - 5 = 0 \to z = \frac{-7 \pm \sqrt{69}}{2} \xrightarrow{\text{چون}} \frac{-7 - \sqrt{69}}{2}$ غ ق ق

$\Delta = 49 + 20 = 69$

$x^2 = \frac{-7 + \sqrt{69}}{2} \to x_1 = \sqrt{\frac{7 + \sqrt{69}}{2}}$ و $x_2 = -\sqrt{\frac{7+\sqrt{69}}{2}} \Rightarrow 2P^2 - 3SP + 2S = 2P^2 =$

$x_1 + x_2 = S = 0$

$2\left(\sqrt{\frac{7+\sqrt{69}}{2}} \times -\sqrt{\frac{7+\sqrt{69}}{2}}\right)^2 = 59 + 7\sqrt{69}$

۶-

$\alpha = \frac{1}{2} + \alpha'$

$\beta = \frac{1}{2} + \beta' \Rightarrow \alpha \cdot \beta = (\alpha' + \frac{1}{2})(\beta' + \frac{1}{2}) = \frac{1}{4} + (\alpha' + \beta')\frac{1}{2} + \alpha'\beta' = -3 = \frac{b}{2}$

($\alpha' + \beta' = -\frac{1}{2}$ ، $\alpha'\beta' = -3$)

$\alpha + \beta = \frac{1}{2} + \alpha' + \frac{1}{2}\beta' = 1 + \alpha' + \beta' = \frac{1}{2} = \frac{a}{2}$

$\frac{-a}{2a} = -\frac{1}{2}$

$\left[\frac{a}{2} \times \frac{b}{2}\right] = \left[\frac{1}{2} \times -3\right] = -2$

-۷

$۲۵(۲\beta^۲+\alpha^۲-\beta)=۱۷$ $\left\{\begin{array}{l}\alpha+\beta=\frac{a}{a}=۱\\ \alpha\cdot\beta=-\frac{b}{a}\end{array}\right.$ $\longrightarrow$ $۲۵\left(\underbrace{\alpha^۲+\beta^۲}_{S^۲-۲P=۱+\frac{۲b}{a}}+\underbrace{\beta^۲-\beta}_{\frac{b}{a}}\right)=۱۷ \rightarrow \frac{\beta^۲-\beta}{a}=-\frac{۱}{۲۵} \Rightarrow ۲۵\beta^۲-۲۵\beta+۱=۰$

ازطرفی: $a\beta^۲-a\beta-b=۰$

$\beta^۲-\beta=\frac{b}{a}$

$\frac{\sqrt{\Delta}}{|a|}=\frac{\sqrt{۶۰۰-۱۰}}{۲۵}=\frac{۲۷\sqrt{۵}}{۵}$

-۸

$x^۲-(a+۱)x+a=۰ \Rightarrow \frac{\sqrt{\Delta}}{|a|}=۲ \rightarrow \frac{\sqrt{a^۲+۲a+۱-۴a}}{۱}=۲ \rightarrow \sqrt{\underbrace{a^۲-۲a+۱}_{(a-۱)^۲}}=۲ \rightarrow |a-۱|=۲$

$a=۳$

a نمی‌تواند ۳- باشد، زیرا آن وقت ریشه‌ها صحیح نمی‌شد

$\Downarrow$

$\frac{x^۲}{۱۰}-\frac{(۲a+۱)}{۱۰}x+b=۰ \Rightarrow \frac{\sqrt{\Delta}}{|a|}=۲ \rightarrow \sqrt{۱۰۰-۴b}=۲ \rightarrow ۲\sqrt{۲۵-b}=۲$

$b=۲۴ \Longrightarrow b-a=۲۴-۳=\boxed{۲۱}$

-۹

کوچکترین مقدار $\rightarrow$ $m^۲=۰ \rightarrow x^۲+x-۱=۰$

$m^۲$ چون ثبت هست و صفر است

$\alpha^۲+\beta^۲=S^۲-۲P=۱+۲=۳$

-۱۰

$y=ax^۲+bx+c$ $\xrightarrow{B\,و\,A}$ $۹a+۳b+c=۲۵a-۵b+c$

$\overline{a+۱}$

$۱۴a=۸b \rightarrow b=۲a$

$\left|\begin{array}{l}-\frac{b}{۲a}=-۱\\ -\frac{\Delta}{۴a}=۱\end{array}\right.$ ازطرفی $\Longrightarrow y=a(x-h)^۲+k$

$\left.\begin{array}{l}y=ax^۲+۲ax+a+۱\\ \alpha^۲+\beta^۲=۵=S^۲-۲P\end{array}\right\} \rightarrow (-۲)^۲-۲\left(\frac{a+۱}{a}\right)=۵$

$a=-\frac{۲}{۳} \rightarrow c=a+۱=-\frac{۲}{۳}+۱=\frac{۱}{۳}$