نام و نام خانوادگی ....هلیا عطایی.... **پاسخنامه تشریحی تکلیف شماره** ..۱۹.... **کلاس** ....یازدهم تجربی C....

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$3x^2 - ax + 4 = 0$, $3\beta = \alpha$

$\alpha\cdot\beta = \frac{c}{a} = \frac{4}{3} \to \alpha\beta = 3\beta(\beta) = 3\beta^2 = \frac{4}{3}$

$\to \beta^2 = \frac{4}{9}$
- (+) $\beta = \frac{2}{3}$, $\alpha = 2$
- (−) $\beta = -\frac{2}{3}$, $\alpha = -2$

$\xrightarrow{x=2} 3(4) - a(2) + 4 = 0$ ، $16 - 2a = 0$ ، $a = 8$

$\xrightarrow{x=-2} 3(4) - a(-2) + 4 = 0 \to 12 + 2a + 4 = 0$ ، $a = -8$

اختلاف دو مقدار a: $(16 = 8 - (-8))$

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$36x^2 - (m+14)x + 1 = 0$ ، $\alpha+\beta = \frac{-b}{a} = \frac{m+14}{36}$

$\sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = 5$

$\downarrow ()^2$

$\frac{1}{\alpha} + \frac{1}{\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = 25$

$\alpha\cdot\beta = \frac{c}{a} = \frac{1}{36} \to \frac{1}{\alpha\beta} = 36$

$\frac{\beta+\alpha}{\alpha\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = 25 \to \frac{m+14}{36}\times 36 + 2\sqrt{36} = 25 \to m + 14 + 12 = 25 \to m = -1$

$mx^2 + 3x + 2 = 0$

$\frac{c}{a} = \alpha'\beta'$ = حاصل‌ضرب ریشه‌ها

$\alpha'\beta' = \frac{c}{a} = \frac{2}{m} = \frac{2}{-1} = -2 \to$ جواب

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$4x^3 + kx^2 - 9x - 2 = 0$ ، $\alpha+\beta = 1$ ، $\alpha\beta = -2$ ، $k = ?$

$\beta = \frac{-2}{\alpha}$

$\alpha + \frac{-2}{\alpha} = 1 \xrightarrow{\times(\alpha)} \alpha^2 - \alpha - 2 = 0 \to \alpha = -1$ ، $\alpha = 2 = -\frac{c}{a}$ ($-2+1=-1$)

$\xrightarrow{x=-1} -4 + k + 9 - 2 = 0 \to k = -3$

$\xrightarrow{x=2} 4(8) + k(4) - 9(2) - 2 = 0$ ، $4k = -12 \to k = -3$

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$x^2 + 6x + a = 0$ ، $3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 85$ ، $\alpha < \beta < 0$ ، $a = ? \to a = 1$

($3\alpha^2 \to 2.5\alpha^2 + 0.5\alpha^2$ ، $2\beta^2 \to 2.5\beta^2 - 0.5\beta^2$)

$\alpha+\beta = \frac{-b}{a} = -6$

$\alpha\cdot\beta = \frac{c}{a} = a$ ، $a > 0$

$\alpha^2 + \beta^2 = S^2 - 2P = 36 - 2a$

$\alpha^2 - \beta^2 = (\alpha+\beta)(\alpha-\beta) = -6(2\sqrt{9-a}) = -12\sqrt{9-a}$

$\alpha - \beta = \frac{\sqrt{\Delta}}{|a|} = \sqrt{36 - 4a} = 2\sqrt{9-a}$

$2.5\alpha^2 + 2.5\beta^2 + 0.5\alpha^2 - 0.5\beta^2 = 12\sqrt{2} + 85$

$2.5(36 - 2a) + 0.5(-12\sqrt{9-a}) = 12\sqrt{2} + 85 \to 90 - 5a - 6\sqrt{9-a} = 12\sqrt{2} + 85$

$5 - 12\sqrt{2} - 5a = 6\sqrt{9-a} \xrightarrow{()^2} 25 + 288 - 120\sqrt{2} + 25a^2 - 50a + 120\sqrt{2} = 324 - 36a$

$25a^2 - 14a - 11 = 0$

- $a = 1$ ق‌ق ✓
- $a = -\frac{11}{25}$ غ‌ق ($\alpha\cdot\beta = a > 0$)

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$x^4 - 7x^2 - 5 = 0$ ، $A = 2P^2 - 3SP + 2S^3 = ?$

$x^2 = t \to t^2 - 7t - 5 = 0 \xrightarrow{\Delta = b^2 - 4ac} \Delta = 49 - 4(1)(-5) \to \Delta = 69$

$t = \frac{-b \pm \sqrt{\Delta}}{2a} \to t = \frac{7 \pm \sqrt{69}}{2}$

- $t = \frac{7 - \sqrt{69}}{2}$ غ‌ق ($t = x^2$) نمی‌تواند منفی باشد
- $t = \frac{7 + \sqrt{69}}{2} = x^2$ ق‌ق $\to x = \pm\sqrt{\frac{7+\sqrt{69}}{2}} \to$ جمع $S = \sqrt{\frac{7+\sqrt{69}}{2}} - \sqrt{\frac{7+\sqrt{69}}{2}} = 0 \to S = 0$

$P = \sqrt{\frac{7+\sqrt{69}}{2}} \times -\sqrt{\frac{7+\sqrt{69}}{2}} = -\frac{7+\sqrt{69}}{2}$

$\to 2\left(\frac{-7-\sqrt{69}}{2}\right)^2 = 2\left(\frac{49+69+14\sqrt{69}}{4}\right)$

$A = 59 + 7\sqrt{69}$

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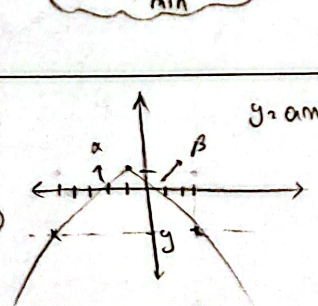
$2x^2 - ax + b = 0 \to \alpha,\beta$ $\quad 2ax^2 + ax - 4 = 0 \to \alpha',\beta'$ $\quad \left[\frac{ab}{4}\right] = -2$ $\alpha+\beta = \frac{a}{2}$ $\quad \alpha'+\beta' = -\frac{a}{2}$ $\alpha\cdot\beta = \frac{b}{2}$ $\quad \alpha'\cdot\beta' = -2$ $\alpha = \frac{1}{2} + \alpha'$, $\beta = \frac{1}{2} + \beta'$ $\to \underbrace{\alpha+\beta}_{\frac{a}{2}} = 1 + \underbrace{\alpha'+\beta'}_{-\frac{a}{2}} \to \frac{2a}{2} = 1 \to a = 1$ $\alpha\beta = \alpha'\beta' + \frac{1}{2}(\alpha'+\beta') + \frac{1}{4} \to \frac{b}{2} = -2 + \frac{1}{2}\left(-\frac{a}{2}\right) + \frac{1}{4} \to b = -4$ (۵) $\left[\frac{ab}{4}\right] = \left[\frac{-4}{4}\right] = \left[-\frac{2}{2}\right] = [-1, 0] = -2$ ← جواب	۶
اختلاف ریشه‌های این معادله؟ $ax^2 - ax - b = 0 \to \alpha,\beta$ ریشه‌هاشانند $\quad 20\beta^3 + 20\alpha^2 - 20\beta = 17$ $\alpha+\beta = \frac{-b}{a} = 1$, $\alpha\beta = \frac{c}{a} = \frac{-b}{a} \to \alpha^2+\beta^2 = S^2 - 2P = 1 - 2\left(-\frac{b}{a}\right) = 1 + \frac{2b}{a}$ $x = \beta \to a(\beta^2 - \beta) = b \to \beta^2 - \beta = \frac{b}{a}$ $20\beta^3 + 20\alpha^2 + 20\beta^2 - 20\beta = 17 \to 20\left((\beta^2+\alpha^2) + (\beta^2 - \beta)\right) = 17$ $1 + \frac{2b}{a} + \frac{b}{a} = \frac{17}{20} \to \frac{3b}{a} = \frac{17-20}{20} = \frac{-3}{20} \to \frac{b}{a} = -\frac{1}{20} \to b = -\frac{a}{20}$ (۲) $ax^2 - ax + \frac{a}{20} = 0$ $\quad (\Delta = b^2 - 4ac) \to \Delta = \frac{4a^2}{5}$ اختلاف ریشه $\lvert\alpha - \beta\rvert = \frac{\sqrt{\Delta}}{\lvert a\rvert} \Rightarrow \lvert\alpha - \beta\rvert = \frac{\sqrt{\frac{4a^2}{5}}}{\lvert a\rvert} = \frac{2\lvert a\rvert}{\sqrt{5}} \times \frac{1}{\lvert a\rvert} = \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ → اختلاف ریشه	۷
$x^2 - (a+1)x + a = 0$ $\quad$ α و β دو فرد متوالی $\alpha = 2k+1$, $\beta = 2k+3$ $\alpha+\beta = \frac{-b}{a} = a+1 \to 2k+1+2k+3 = a+1 \to 4k = a-3$ $\alpha\cdot\beta = \frac{c}{a} = a \to (2k+1)(2k+3) = 4k^2+8k+3 = a$ $\to 4k^2 + 2(a-3) + 3 = a \to 4k^2 + 2a - 6 + 3 = a$ $\to 4k^2 + 4k = 0 \to k^2 + k = 0 \to k(k+1) = 0 \to \begin{cases} k = 0 \to 1, 3 \text{ فرد} \to a = 3 \checkmark \\ k = -1 \to -1, 1 \to a = -1 \end{cases}$ $x^2 - (3a+1)x + b = 0$ $\quad$ α' و β' دو عدد زوج متوالی $= 2t, 2t+2$ $\alpha'+\beta' = \frac{-b}{a} = 3a+1$ $\Delta P = \lvert\alpha\beta - \alpha'\beta'\rvert = \lvert a - b\rvert = 24 - 3 = 21$ جواب $2t + 2t + 2 = 4t + 2 = 3a + 2 \to 4t = 3a - 1$ $\alpha'\beta' = b \to 4t^2 + 4t = b$ $a = 3 \to 4t = 8 \to t = 2$ زوج $\quad$ (۱) $a = -1 \to 4t = -4 \to t = -1$ فرد $4(4) + 4(2) = b \to 24 = b$ $\quad$ $4 - 4 = b = 0$ چون $b = \beta'\alpha' = 0$ درنمیاد $\lvert b - a\rvert = 24 - 3 = 21$	۸
$x^2 - x - 1 - m^2 = 0$ $\quad \alpha^2+\beta^2 = ?$ (min) $\alpha+\beta = \frac{-b}{a} = \frac{1}{1} = 1$ $\alpha\cdot\beta = \frac{c}{a} = -m^2-1$ (۵) $\alpha^2+\beta^2 = S^2 - 2P = 1 - 2(-m^2-1) = 1 + 2m^2 + 2 \to \alpha^2+\beta^2 = 3 + 2m^2$ $m^2_{\text{کمترین مقدار}} = 0 \to m = 0 \to (\alpha^2+\beta^2)_{min} = 3$ $m_{min} = \frac{-b}{2a} = 0$	۹
$x_{min} = \frac{-3-1}{2} = \frac{-2}{2} = -1$ $y_{min} = 1$  $x_{min} = \frac{-b}{2a} = -1 \to b = 2a$ $y_{min} = -\frac{\Delta}{4a} = \frac{-b^2 + 4ac}{4a} \xrightarrow{b = 2a} \frac{-4a^2 + 4ac}{4a} = 1 \to -a + c = 1 \to c = a+1$ $C = a+1 \to C = -\frac{2}{3} + \frac{3}{3} = \frac{1}{3} \to$ عرض از مبدأ $\to$ با عرض $\frac{1}{3}$ محور y را قطع می‌کند $y = ax^2 + bx + c \to y = ax^2 + 2ax + a + 1$ $\quad \alpha^2+\beta^2 = 5$ $\alpha+\beta = \frac{-b}{a} = \frac{-2a}{a} = -2$ $\alpha\cdot\beta = \frac{c}{a} = \frac{a+1}{a}$ (۵) $\alpha^2+\beta^2 = S^2 - 2P = 4 - 2\left(\frac{a+1}{a}\right) = 5$ $\to \frac{a+1}{a} = -\frac{1}{2} \to 2a + 2 = -a \to 3a = -2 \to a = -\frac{2}{3}$	۱۰