نام و نام خانوادگی ......دنیا داداش نصیری...... پاسخنامه تشریحی تکلیف شماره ...۱۹... کلاس ......

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$3x^2 - ax - 4 = 0$

ریشه‌ها $\to \alpha , 3\alpha$

ضرب ریشه‌ها $= 3\alpha \times \alpha = \frac{4}{3} \to 3\alpha^2 = \frac{4}{3} \to \alpha^2 = \frac{4}{9} \to \alpha = \pm\frac{2}{3}$

$\alpha = \frac{2}{3} , 3\alpha = 2$ جمع $= 2 + \frac{2}{3} = \frac{8}{3} = \frac{a}{3} \to a = 8$

$\alpha = -\frac{2}{3} , 3\alpha = -2$ جمع $= -2 - \frac{2}{3} = -\frac{8}{3} = \frac{a}{3} \to a = -8$

اختلاف $a = 8 - (-8) = 16$

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$36x^2 - (m+14)x + 1 = 0$ ریشه‌ها $\to \alpha , \beta$

$\sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = 5 \xrightarrow{\text{به توان ۲}} \frac{1}{\alpha} + \frac{1}{\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = 25$

$\frac{\alpha+\beta}{\alpha\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = 25 \to \frac{\frac{m+14}{36}}{\frac{1}{36}} + 2\sqrt{\frac{1}{\frac{1}{36}}} = 25 \to m + 14 + 2\times 6 = 25 \to m = -1$

$mx^2 + 3x + 2 = 0 \to -1x^2 + 3x + 2 = 0 \to$ ضرب $= -2$

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$4x^3 + kx^2 - 9x - 2 = 0$

$\alpha + \beta = 1 \to \alpha + (-\frac{2}{\alpha}) = 1 \to \frac{\alpha^2 - 2}{\alpha} = 1 \to \alpha^2 - \alpha - 2 = 0 \to (\alpha - 2)(\alpha + 1) = 0$

$\alpha\beta = -2 \to \beta = -\frac{2}{\alpha}$

$\beta = -1 \leftarrow \alpha = 2$ ، $\alpha = -1 \to \beta = 2$

$\xrightarrow{x=2} \cancel{4\times 8}_{32} + 4k - 18 - 2 = 0 \to 4k = -12$

$k = -3$

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$x^2 + 6x + a = 0$ $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (-6)^2 - 2(a) = 36 - 2a$ $|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{36-4a}}{1}$

$3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 85 \to \frac{5}{2}\alpha^2 + \frac{5}{2}\beta^2 + \frac{1}{2}\alpha^2 - \frac{1}{2}\beta^2 = \frac{5}{2}(\alpha^2+\beta^2) + \frac{1}{2}(\alpha-\beta)(\alpha+\beta) = 85 + 12\sqrt{2}$

$\frac{5}{2}(36 - 2a) + \frac{1}{2}(-6)(-\sqrt{36-4a}) = 85 + 12\sqrt{2} \to 90 - 5a + 3\sqrt{36-4a} = 85 + 12\sqrt{2}$

$90 - 5a = 85 \to a = 1$

$3\sqrt{36-4a} = 12\sqrt{2} \xrightarrow{a=1} 3\sqrt{36-4} = 3\underbrace{\sqrt{32}}_{4\sqrt{2}} = 12\sqrt{2}$

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$x^4 - 7x^2 - 5 = 0 \xrightarrow{x^2 = t} t^2 - 7t - 5 = 0 \to t = \frac{7 \pm \sqrt{69}}{2} \to x^2 = \frac{7 - \sqrt{69}}{2}$ ✗

$x^2 = \frac{7 + \sqrt{69}}{2} \to x = \pm\sqrt{\frac{7+\sqrt{69}}{2}} \to S = 0 , P = \frac{-7-\sqrt{69}}{2}$

$2P^2 - SP + 2S$

$= 2\times\left(\frac{-7-\sqrt{69}}{2}\right)^2 - 2(0)\left(\frac{-\sqrt{69}-7}{2}\right) + 2\times 0 =$

$\frac{49 + 69 + 14\sqrt{69}}{2} = \frac{118 + 14\sqrt{69}}{2} = 59 + 7\sqrt{69}$

$2x^2 - ax + b = 0 \to$ ریشه‌ها $= \alpha + \frac{1}{2}, \beta + \frac{1}{2} \to (\alpha+\frac{1}{2})(\beta+\frac{1}{2}) = \alpha\beta + \frac{1}{4} + \frac{1}{2}(\alpha+\beta) = \frac{b}{2}$ $2ax^2 + ax - 6 = 0 \to$ ریشه‌ها $= \alpha, \beta \to \alpha\beta = -3$; $\to \alpha+\beta+1 = \frac{a}{2} \to a = 1$ $\alpha+\beta = \frac{-a}{2a} = -\frac{1}{2}$ $-3 + \frac{1}{4} + \frac{1}{2}(-\frac{1}{2}) = \frac{b}{2}$ $b = -6$ $\left[\frac{ab}{4}\right] = \left[\frac{-6 \times 1}{4}\right] = -2$	۶
$ax^2 - ax - b = 0 \to a\alpha^2 - a\alpha - b = 0 \to \alpha^2 = \frac{a\alpha + b}{a}$ $40\beta^2 + 20\alpha^2 - 20\beta = 17 \to \beta^2 = \frac{a\beta + b}{a}$ $\frac{40(a\beta+b)}{a} + \frac{20(a\alpha+b)}{a} - 20\beta = 17 \to 17a = 40(a\beta+b) + 20(a\alpha+b) - 20a\beta = 17$ $40a\beta + 40b + 20a\alpha + a - 20a\beta = 17 \to \underbrace{20a\beta - 20a\beta}_{0} + 40b + 20a = 17 \to b = -\frac{3}{40}a$ $20a(\underbrace{2\beta+\alpha}_{\beta+\beta+\alpha}) = 20a\beta$ اختلاف ریشه‌ها $= \frac{\sqrt{\Delta}}{\lvert a \rvert} = \frac{\sqrt{a^2+4ab}}{\lvert a \rvert} = \frac{\sqrt{a^2 - \frac{3}{10}a^2}}{\lvert a \rvert} = \frac{\sqrt{\frac{7}{10} \times a^2}}{\lvert a \rvert} = \frac{\sqrt{7}}{\sqrt{10}}$	۷
$x^2 - (a+1)x + a = 0 \xrightarrow{\text{ریشه‌ها}} \alpha, \alpha+2 \to 2\alpha+2 = a+1$ ، $a = \alpha^2 + 2\alpha \to 2\alpha + 2 - 1 = \alpha^2 + 2\alpha \to \alpha = \pm 1$ $x^2 - (3a+1)x + b = 0 \xrightarrow{\text{ریشه‌ها}} \beta, \beta+2$ ؛ α طبیعی درنظر ← $\alpha = 1 \leftarrow a = 3$ $2\beta + 2 = 3a + 1 = 10 \to 2\beta = 8 \to \beta = 4$ $(4)(6) = 24 = +b \to b = +24$ اولی $P = 3$ دومی $P = 24 \to 24 - 3 = 21$	۸
$x^2 + x - 1 - m^2 = 0 \to \alpha^2 = m^2 + 1 - \alpha$ ، $\beta^2 = m^2 + 1 - \beta$ $\alpha^2 + \beta^2$ $\to m^2 + 1 - \alpha + m^2 + 1 - \beta = m^2 + 2 - \underbrace{(\alpha+\beta)}_{-1} = 2m^2 + 3 \to x_{min} = \frac{-b}{2a} = 0$ مقدار min $= 3$	۹
$A(3, y)$ ، $B(-5, y)$ طول نقطه وسط $\frac{3-5}{2} = -1$ طول نقطه جدید $= -1$ ؛ عرض " " $= 1$ جدید $\xrightarrow{x-(-1)} a(x+1)^2 + 1 = ax^2 + a + 2ax + 1 = ax^2 + 2ax + a + 1$ $\alpha^2 + \beta^2 = 5$ $\alpha^2 + \beta^2 = S^2 - 2P = \left(\frac{-2a}{a}\right)^2 - 2\left(\frac{a+1}{a}\right) = 4 - 2\frac{(a+1)}{a} = 5$ $4a - 2a - 2 = 5a \to 3a = -2$ ، $a = -\frac{2}{3}$ $-\frac{2}{3}x^2 + \left(-\frac{4}{3}\right)x + \frac{1}{3} = 0$ عرض از مبدا	۱۰