Subject:

Year. Month. Date. ( )

دریا دیانی دردستم

①

$3x^2 - ax + 4 = 0$

ریشه‌ها $\Rightarrow \beta , 3\beta$ (x_1, x_2) $\Rightarrow S = 4\beta = \frac{a}{3}$

$\beta = \frac{2}{3} \Rightarrow a = 12\beta \Rightarrow 12 \times \frac{2}{3} = 8$

$\beta = -\frac{2}{3} \Rightarrow a = 12\beta \Rightarrow 12 \times -\frac{2}{3} = -8$

$P = 3\beta^2 = \frac{4}{3} \rightarrow \beta^2 = \frac{4}{9} \rightarrow \beta = \pm\frac{2}{3}$

$a_1 - a_2 = 16$

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②

$36x^2 - (m+14)x + 1 = 0$

$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = 5 \Rightarrow \left(\frac{\sqrt{\beta}+\sqrt{\alpha}}{\sqrt{\alpha\beta}} = 5\right)^2 \rightarrow \frac{\alpha+\beta+2\sqrt{\alpha\beta}}{\alpha\beta} = 25$ $\qquad S = -\frac{b}{a}$, $P = \frac{c}{a}$

$\frac{S+2\sqrt{P}}{P} = \frac{\frac{m+14}{36} + 2\sqrt{\frac{1}{36}}}{\frac{1}{36}} = 36 \times \left(\frac{m+14}{36} + \frac{1}{3}\right) = m+14+12 = 25 \rightarrow m = -1$

$mx^2 + 3x + 2 = 0 \rightarrow -x^2 + 3x + 2 \rightarrow P = \frac{c}{a} = \frac{2}{-1} = -2$

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③

$4x^3 + kx^2 + 9x - 2 = 0$

$\left.\begin{matrix}\alpha+\beta = 1 \\ \alpha\beta = -2\end{matrix}\right\} (\alpha+\beta)^2 = 1 \Rightarrow \alpha^2+\beta^2+2\alpha\beta = 1 \Rightarrow \alpha^2+\beta^2 - 4 = 1 \rightarrow \left.\begin{matrix}\alpha^2+\beta^2 = 5 \\ \alpha+\beta = 1\end{matrix}\right\} \begin{matrix}\beta = -1 \\ \alpha = 2\end{matrix}$

$4 \times (-1)^3 + k + 9 - 2 = 0 \Rightarrow k = -3$

$4 \times 8 + 4k - 18 - 2 = 0 \Rightarrow k = -3$

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④

$x^2 + 6x + a = 0$ $\qquad S = -\frac{b}{a} = \frac{-6}{1} = -6$ $\qquad (\alpha-\beta)(\alpha+\beta)$

$\alpha < \beta < 0$ $\qquad P = \frac{c}{a} = a$

$3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 85 \Rightarrow 2.5\alpha^2 + 2.5\beta^2 + \frac{1}{2}\alpha^2 - \frac{1}{2}\beta^2 \Rightarrow 2.5(\alpha^2+\beta^2) + \frac{1}{2}(\alpha^2-\beta^2)$

$2(\alpha^2+\beta^2) + \alpha^2 = 2(S^2-2P) + \alpha^2 \Rightarrow 2(36-2a) +$ $\qquad 36-2a$

$\alpha^2 - \beta^2 = (\alpha-\beta)\underset{-6}{(\alpha+\beta)} \Rightarrow |\alpha-\beta| = \frac{\sqrt{\Delta}}{|a|} \Rightarrow \alpha<\beta \Rightarrow -\sqrt{\Delta}$

$\frac{5}{2}(\alpha^2+\beta^2) + \frac{1}{2}(\alpha^2-\beta^2) \Rightarrow 90 - 5a + 3\sqrt{36-4a} = 12\sqrt{2} + 85$ $\qquad 90 - 5a = 85$, $\boxed{a = 1}$

$3\sqrt{36-4a} \Rightarrow \boxed{a = 1}$

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⑤

$x^4 - 7x^2 - 5 = 0 \Rightarrow t^2 - 7t - 5 = 0 \rightarrow x_1^2 = \frac{-b+\sqrt{\Delta}}{2a} \Rightarrow x_1^2 = \frac{-7+\sqrt{69}}{4}$

S, P

$x_2^2 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{+7-\sqrt{69}}{4} \rightarrow$ منفی غ ق

$x_1^2 = \frac{-7+\sqrt{69}}{4}$ $\quad \left.\begin{matrix} x_1 = +\sqrt{\frac{-7+\sqrt{69}}{4}} \\ x_2 = -\sqrt{\frac{-7+\sqrt{69}}{4}} \end{matrix}\right\} S = 0 \quad P = \frac{-7+\sqrt{69}}{4}$

$2P^2 - 3SP + 5S \Rightarrow 2P^2 = 2 \times \left(\frac{-7+\sqrt{69}}{4}\right)^2 \Rightarrow \frac{49+69+14\sqrt{69}}{4} \times 2 = \boxed{59 + 7\sqrt{69}}$

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۴) $2x^2 - ax + b = 0 \rightarrow \alpha + \frac{1}{2} + \beta + \frac{1}{2} = \frac{a}{2} \Rightarrow \alpha + \beta + 1 = \frac{a}{2}$

$\alpha + \beta = -\frac{1}{2} \Rightarrow \frac{a-2}{2} = -\frac{1}{2} \Rightarrow a = 1$

$\alpha\beta = \frac{-3}{a}$ $\qquad (\frac{1}{2} + \alpha)(\frac{1}{2} + \beta) = \frac{1}{4} + \underbrace{(\alpha+\beta)}_{-\frac{1}{2}}\frac{1}{2} + \underbrace{\alpha\beta}_{-2} = \frac{b}{2}$

$\frac{1}{4} - \frac{1}{4} - \frac{12}{4} = \frac{b}{2} \Rightarrow b = -6$ $\qquad \left[\frac{ab}{4}\right] = \left[\frac{-6}{4}\right] = \left[-\frac{3}{2}\right] = -2$

۷) $ax^2 - ax - b = 0 \qquad S = 1 \qquad P = -\frac{b}{a}$

$20\beta^2 + 20\alpha^2 - 20\beta = 17 \longrightarrow 20(\beta^2 - \beta) + 20\beta^2 + 20\alpha^2$

$20(\frac{b}{a})$ $\quad 20(\alpha^2 + \beta^2)$

$20(S^2 - 2P)$

$20(1 + 2\frac{b}{a})$

$\beta^2 - \beta = \frac{b}{a}$

$20 + 40\frac{b}{a} + 20\frac{b}{a} = 20 + 60\frac{b}{a} = 17 \rightarrow \frac{b}{a} = -\frac{1}{20}$

$b = -\frac{1}{20}a \Rightarrow \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{a^2 + 4ab}}{|a|} = \frac{\sqrt{a^2 - \frac{a^2}{5}}}{|a|} = \frac{|a|\sqrt{\frac{4}{5}}}{|a|} = \frac{2}{\sqrt{5}}$

۸) $x^2 - (a+1)x + a = 0$

$2k+1, 2k-1 \rightarrow a = (2k+1)(2k-1) = 4k^2 - 1$

$a + 1 = 4k \Rightarrow a = 4k - 1$

$4k^2 - 4k = 0 \rightarrow k = 0, 1$ $\qquad a = -1$

$b = 4k'^2 + 4k' \Rightarrow 2a + 1 = 4k' + 2 \Rightarrow 10 = 4k' + 2 \Rightarrow k' = 2 \quad b = 24$

$x^2 - (2a+1)x + b = 0$ $\qquad |a - b| = 21$

۹) $x^2 + x - 1 - m^2 = 0$

$\alpha^2 + \beta^2 = S^2 - 2P = (-1)^2 - 2\left(\frac{-1-m^2}{1}\right) = 1 + 2 + 2m^2 = 2m^2 + 3$

$\alpha\beta = -1 - m^2$ $\qquad 2 + 2m^2 \longrightarrow \min \Big|_{m=0}$ $\qquad$ کمترین $= 3$

۱۰) $A(2, y) \quad B(-2, y)$

$\text{عمود} \Big| \frac{-b}{2a} = -1 \qquad \frac{2-2}{2} = -1 \Rightarrow a(x+1)^2 + b = ax^2 + a + 2ax + 1$

عرض رأس $\Rightarrow 1 - \frac{2}{2} = \frac{1}{2}$

$\alpha + \beta = -\frac{2a}{a} = -2 \qquad \alpha\beta = \frac{a+1}{a}$

$(\alpha+\beta)^2 = 4 = \frac{\alpha^2 + \beta^2}{a} + 2\alpha\beta \Rightarrow \alpha\beta = -\frac{1}{2} = \frac{a+1}{a} \Rightarrow a = -\frac{2}{3}$

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