نیوشا هاشم پور — تکلیف ۱۹ — یازدهم تجربی C

۱

$3x^2 - ax + 4 = 0$ $\quad x_1 = 3x_2 \quad x_1x_2 = \frac{4}{3} \quad 3x_2^2 = \frac{4}{3} \to x_2^2 = \frac{4}{9}$

$x_2 = \pm\frac{2}{3} \to x_1 + x_2 = \frac{a}{3} \to 3x_2 + x_2 = 4x_2 = \frac{8}{3} = \frac{a}{3} \to \boxed{a=8}$

$4x_2 = -\frac{8}{3} = \frac{a}{3} \to \boxed{a=-8} \qquad 8-(-8) = \boxed{16}$

۲

$36x^2 - (m+14)x + 10 = 0 \to x_1 + x_2 = \frac{m+14}{36} \quad x_1x_2 = \frac{5}{18}$

$\sqrt{\frac{1}{x_2}} + \sqrt{\frac{1}{x_1}} = 5 \xrightarrow{\text{به توان ۲}} \frac{1}{x_1} + \frac{1}{x_2} + 2\sqrt{\frac{1}{x_1x_2}} = 25 \to \frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1+x_2}{x_1x_2}$

$\xrightarrow{\text{جایگذاری}} \frac{\frac{m+14}{36}}{\frac{5}{18}} + 2\sqrt{\frac{18}{5}} = 25 \to m = 236 - 20\sqrt{\frac{18}{5}}$

(معادله دوم $\to P \to \frac{2}{m} \to \frac{2}{236 - 20\sqrt{\frac{18}{5}}}$

۳

$\alpha+\beta+\gamma = -\frac{k}{4} \to \alpha\beta + \alpha\gamma + \beta\gamma = -\frac{9}{4} \quad \alpha\beta\gamma = \frac{1}{2}$

$\alpha+\beta = 1 \quad \alpha\beta = -2 \to \alpha\beta\gamma = \frac{1}{2} \quad (-2)\gamma = \frac{1}{2} \to \gamma = -\frac{1}{4}$

$\alpha+\beta+\gamma = 1 - \frac{1}{4} = \frac{3}{4} \to \alpha+\beta+\gamma = -\frac{k}{4} \to \boxed{k=-3}$

۴

$x^2 + 6x + a = 0 \to \alpha = -3 - \sqrt{9-a}\ ,\ \beta = -3 + \sqrt{9-a}$

$\alpha^2 = (-3-\sqrt{9-a})^2 = 18 - a + 6\sqrt{9-a} \quad \beta^2 = 18 - a - 6\sqrt{9-a}$

$3(18 - a + 6\sqrt{9-a}) + 2(18 - a - 6\sqrt{9-a}) = 12\sqrt{2} + 85 \to$

$90 - 5a + 6\sqrt{9-a} = 12\sqrt{2} + 85 \to 6\sqrt{9-a} = 5a + 12\sqrt{2} - 5 \to \boxed{a=1}$

۵

$y = x^2 \quad y^2 - 7y - 5 = 0 \to y = \frac{7 \pm \sqrt{49+20}}{2} = \frac{7 \pm \sqrt{69}}{2}$

$y = \frac{7+\sqrt{69}}{2} \to x = \pm\sqrt{y} \quad S = \sqrt{y} + (-\sqrt{y}) = 0 \quad P = (\sqrt{y})(-\sqrt{y}) = -y = -\frac{7+\sqrt{69}}{2}$

(چون $x^2 \geq 0$)

$2P^2 \to P^2 = y^2 \to y^2 = 7y + 5 \to 2P^2 = 2(7y+5) = 14y + 10$

$14 \times \frac{7+\sqrt{69}}{2} + 10 = 49 + 7\sqrt{69} + 10 \to \boxed{59 + 7\sqrt{69}}$

(۶)

$r_1+\frac{1}{2} , r_2+\frac{1}{2}$ $\quad 2ax^2+ax-4=0 \to r_1+r_2=\frac{-a}{2a}=-\frac{1}{2}$

$r_1+r_2=-\frac{a}{2a}=-\frac{1}{2}$ و $r_1r_2=\frac{-4}{2a}=-\frac{2}{a}$ $\quad (r_1+r_2)+1=-\frac{1}{2}+1=\frac{1}{2}$

$\frac{a}{2}=\frac{1}{2} \to a=1$ ضرب ریشه‌ها $\to (r_1+\frac{1}{2})(r_2+\frac{1}{2})=r_1r_2+\frac{1}{2}(r_1r_2)+\frac{1}{4}$

$-\frac{2}{a}+\frac{1}{2}(-\frac{1}{2})+\frac{1}{4}$ $\quad a=1 \to -2-\frac{1}{4}+\frac{1}{4}=-2$

$\frac{b}{2}=-2 \to b=-4$ $\quad |\frac{ab}{4}| \to |\frac{-4}{2}| \to \boxed{-2}$

(۷)

$ax^2-ax-b=0$ $\quad \alpha+\beta=\frac{a}{a}=1$ $\quad \alpha\beta=\frac{-b}{a}$ $\quad \alpha=1-\beta$ $\quad \alpha^2=(1-\beta)^2=1-2\beta+\beta^2$

$40\beta^2+20(1-2\beta+\beta^2)-20\beta=17 \to 40\beta^2+20-40\beta+20\beta^2-20\beta=17$

$60\beta^2-60\beta+20=17 \to 20\beta^2-20\beta+1=0$ $\quad \beta=\frac{20\pm\sqrt{320}}{40}$

$|\beta_1-\beta_2|=\frac{\sqrt{320}}{20} \to \sqrt{320}=8\sqrt{5} \to |\beta_1-\beta_2|=\boxed{\frac{2\sqrt{5}}{5}} \to$ چون $\alpha+\beta=1$

(۸)

$x^2-(a+1)x+a=0 \to \overset{x_2}{\alpha}, \overset{x_1}{\alpha+2} \to \left.\begin{matrix} 2\alpha+2=a+1 \\ a=\alpha^2+2\alpha \end{matrix}\right\} \to \alpha=\pm1$

$x^2-(3a+1)x+b=0 \to \overset{x_2}{\beta}, \overset{x_1}{\beta+2}$ $\quad$ غقق -1 ← $a=3$ و $\alpha=1$

$2\beta+2=3a+1 \to \beta=4$

$4\times6=24=b \to b=24$

P اول $\to 3$ $\quad 24-3=\boxed{21}$

P دوم $\to 24$

(۹)

$x^2+x-1-m^2=0 \to \begin{cases} \alpha^2=m^2+1-\alpha \\ \beta^2=m^2+1-\beta \end{cases}$

$\alpha^2+\beta^2 \to m^2+1-\alpha+m^2+1-\beta=2m^2+2-(\alpha+\beta) \to 2m^2+3 \to x_{min}=\frac{-b}{2a}=0$

$min=3$

(۱۰)

$x=\frac{3+(-5)}{2}=-1$ $\quad (-1,1)$ $\quad y=a(x+1)^2+1 \to 0=a(x+1)^2+1$

$(x+1)^2=-\frac{1}{a} \to x=\alpha, \beta=-1\pm\sqrt{-\frac{1}{a}}$ $\quad \alpha+\beta=-2$ $\quad \alpha\beta=\frac{1}{1}+\frac{1}{a}$ $\quad \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$5=4-2(1+\frac{1}{a}) \to a=-\frac{2}{3}$ $\quad y=a(1)^2+1=-\frac{2}{3}+1=\boxed{\frac{1}{3}}$