۱۴٫۵

سارینا اسپانی _ یازدهم ریاضی _ شماره تکلیف ۱

۱) ۰٫۵

$3x^2 - ax + 4 = 0$

$3\alpha^2 = \frac{4}{3} \to \alpha = \pm\frac{2}{3}$ ، $\alpha = \frac{2}{3} \to \beta = 2 \to a = 8$ ، $\alpha = -\frac{2}{3} \to \beta = -2 \to a = -8$

$8 - (-8) = 16$

$\alpha = x \qquad \beta = 3x$

$\frac{4}{3} = 3\alpha^2 \Rightarrow \frac{\frac{4}{3}}{\frac{3}{1}} = \frac{4}{9} = x^2 \longrightarrow x = \pm\frac{2}{9}$

$+\frac{2}{9} \longrightarrow \frac{+2}{9} + 3\left(\frac{2\times}{3\,9 \times 3}\right)^3 = \frac{8}{9} \qquad a = -\frac{8}{9}$

$-\frac{2}{9} \longrightarrow -\frac{2}{9} + 3\left(\frac{-2}{9}\right) = \frac{-2-6}{9} = \frac{-8}{9} \longrightarrow a = \frac{8}{9}$

اختلاف دو مقدار $\longrightarrow \frac{+8}{9} - \left(-\frac{8}{9}\right) = \frac{16}{9}$

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۲) ۰

$36x^2 - (m+14)x + \beta = 0$

$\alpha + \beta + 2\sqrt{\alpha\beta} = 25 \to \frac{\frac{m+14}{36} + 2\sqrt{\frac{1}{36}}}{\frac{1}{36}} \to m + 14 + 12 = 25 \to m = -1$

$\left(\sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}}\right)^2 = 5 \to -x^2 + 2x + 2 \to \frac{c}{a} = -2$

$\frac{1}{\alpha} + \frac{1}{\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = \frac{\frac{\alpha+\beta}{\alpha\beta}}{} + 2\sqrt{\frac{1}{\frac{1}{36}}} = 5$ ، $\frac{\frac{m+14}{36}}{\frac{1}{36}}$

$m + 14 + 2(6) = 5$

$m + 26 = 5 \longrightarrow m = -21$

$\frac{2}{m} \longrightarrow \frac{2}{-21} \longrightarrow$ حاصلضرب ریشه‌ها

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۳) ۰٫۵

$4x^3 + kx^2 - 9x - 2 = 0 \qquad \alpha\beta = -2 \quad \alpha + \beta = 1$

معادله درجه اولی که ریشه آن α و β باشد $\longrightarrow (x-\alpha)(x-\beta) = x^2 - x - 2$

$-(\alpha+\beta) \qquad -\alpha x - \beta$

$(x-2)(x+1) = 0 \qquad x = 2 \quad x = -1$

$\alpha = 2 \quad \beta = -1 \longrightarrow -4 + k + 9 - 2 = 0$

$k = -3$

$S=-4$ $\quad \alpha\beta=\alpha$ $\quad \alpha,\beta=\frac{-4\pm\sqrt{16-4a}}{2}\longrightarrow \alpha<\beta\longrightarrow \alpha=-2-\sqrt{9-a}$ (۴

$4\alpha^2+2\beta^2=\alpha^2+2(S^2-2P)=12P+8S\longrightarrow \alpha=1$ ۵

$x^4-7x^2-3=0$ $\quad S,P$ ۵

$t^2-7t-3=0$ $\quad t=x^2$

$t=\frac{7\pm\sqrt{49}}{2}\longrightarrow \frac{7-\sqrt{49}}{2}$ ق ق غ

$t\geq 0$ زیرا

$\frac{7+\sqrt{49}}{2}=t\longrightarrow x=\pm\sqrt{\frac{7+\sqrt{49}}{2}}$ ۶ $S=0$ ، $P=\frac{-7-\sqrt{49}}{2}$

$2\left(-\left(\frac{7+\sqrt{49}}{2}\right)\right)^2=2\times\left(\frac{49+49+14\sqrt{49}}{4}\right)=\frac{118+14\sqrt{49}}{2}$

$=59+7\sqrt{49}$

۴) $2x^2 - ax + b = 0$

ریشه‌ها $= a+\frac{1}{2}$ ، $\beta+\frac{1}{2}$

$a+\beta+1 = \frac{a}{2}$

$\frac{-a}{2} \rightarrow a = 1$ ۳

$(a+\frac{1}{2})(\beta+\frac{1}{2})$

$a\beta + \frac{a}{2} + \frac{\beta}{2} + \frac{1}{4}$

$-2 - \frac{1}{4} + \frac{1}{4} = (-2)$

$\frac{b}{2} = -2 \leftarrow b = -4$

$2x^2 + ax - 4 = 0$

ریشه‌ها $\rightarrow \alpha, \beta$ $\quad \alpha.\beta = -2$

$\alpha+\beta = \frac{-a}{2} = \frac{-1}{2}$

$\left[\frac{ab}{4}\right] = \left[\frac{-4}{4}\right] = (-1)$

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۷) $ax^2 - ax - b = 0$

$50\beta^2 + 50\alpha^2 - 20\beta = 17 \rightarrow 50\beta^2 + 20(1-\beta)^2 - 20\beta = 17$

$50\beta^2 + 20 + 20\beta^2 - 50\beta - 20\beta = 17$

$70\beta^2 - 70\beta + 3 = 0$

۳ $\quad 20\beta^2 - 20\beta + 1 = 0$

$\beta = \frac{20 \pm \sqrt{320}}{40} = \frac{5 \pm 2\sqrt{5}}{10}$

$\alpha + \beta = S = -\left(\frac{-a}{a}\right) = 1$

$\alpha = 1 - \beta$

$\alpha = 1 - \beta = 1 - \frac{5-2\sqrt{5}}{10} = \frac{5+2\sqrt{5}}{10} \rightarrow |\alpha - \beta| = \frac{2\sqrt{5}}{5}$

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$\alpha = 1 - \beta = 1 - \frac{5+2\sqrt{5}}{10} = \frac{5-2\sqrt{5}}{10} \Rightarrow \frac{2\sqrt{5}}{5} = |\alpha - \beta|$

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۸) $x^2 - (a+1)x + a = 0$ ← دو عدد فرد متوالی جذب

$2t+1$ ، $2t+3$

$4t+4 = a+1 \Rightarrow a = 4t+3$ ← دو عدد زوج متوالی $4t+2$

$x^2 - (2a+1)x + b = 0$

$4t^2 + 8t + 3 = a$

$4t^2 + 8t + 4 = 4t + 4$

$4t^2 + 4t = 0$

$4t(t+1) = 0$ چون ریشه باید طبیعی باشد

$t = 0 \checkmark$ ، $t = -1$ غ ق ق

۳ $\quad 2k$ ، $2k+2$

$4k+2 = 3\alpha + 1 \rightarrow a = \frac{4k+1}{3}$

$4k^2 + 4k = b$

$\rightarrow a = 3$

$|3 - 4k| = 21$

$\frac{4k+1}{3} = 3$

$k = 2 \leftarrow 4k = 9 - 1$

⑨

$x^2 + x - 1 - m^2 = 0$

$\alpha^2 + \beta^2 = S^2 - 2P = 1 + 2 + 2m^2 = 3 + 2m^2 = 3$

$S = -1$

کمترین مقدار = ۰

$P = \frac{-1 - m^2}{1}$ $\qquad m = 0$

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⑩

$y_S = 1$ $\qquad ax^2 + bx + c$ $\qquad (-5, y)$ و $(3, y)$

$a - b + c = 1$

میانگین طول ها

$x_S = \frac{3 - 5}{2} = -1$

$\alpha^2 + \beta^2 - 5 = S^2 - 2P$

$\rightarrow 4 - 2P = 5 \rightarrow P = \frac{-1}{2}$

$x_S = \frac{-b}{2a} = -1$

$25a - 5b = 9a + 3b$

$\alpha + \beta = \frac{-b}{a}$

(د)

$16a - 8b = 0$

$2x_S = S$

$2\ 16a = 8b$

$\boxed{S = -2}$

$a = \frac{b}{2} \rightarrow \frac{c}{\frac{b}{2}} = \frac{-1}{2} \rightarrow \frac{2c}{b} = \frac{-1}{2}$

$4c = -b$

$\frac{b}{2} - b + c = 1$ $\qquad b = -4c$

$b - 2b + 2c = 2 \rightarrow 2c - b = 2$

$2c + 4c = 2 \rightarrow 6c = 2$

$\boxed{c = \frac{1}{3}}$

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