

هسته با مرکز

تکلیف مسئله ۱۸

پارادوکس دفر

$$\frac{\text{طول}}{\text{عرض}} = \frac{z}{y} = \frac{a}{\varepsilon} \rightarrow y = \varepsilon a, z = a \varepsilon$$

①

$$S_1 = \{k \times a k = 2 \cdot k^2 \quad \frac{z'}{y} = \frac{1 + \sqrt{a}}{2} \rightarrow z' = \varepsilon k (1 + \frac{\sqrt{a}}{2})$$

$$S_2 = \varepsilon k \times \varepsilon k \left(\frac{1 + \sqrt{a}}{2} \right) = 14 k^2 \left(\frac{1 + \sqrt{a}}{2} \right)$$

$$\frac{S_2}{S_1} = \frac{14 k^2 \left(\frac{1 + \sqrt{a}}{2} \right)}{2 \cdot k^2} = \frac{7}{a} (1 + \sqrt{a})$$

$$\frac{\text{قطر}}{\text{طول}} = \frac{1 + \sqrt{a}}{2} \quad \begin{matrix} \text{طول} \rightarrow z \\ \text{عرض} \rightarrow y \end{matrix} \quad \text{قطر} \rightarrow \sqrt{z^2 + y^2}$$

②

$$\frac{\sqrt{z^2 + y^2}}{z} = \frac{1 + \sqrt{a}}{2} \rightarrow \frac{z^2 + y^2}{z^2} = \frac{(1 + \sqrt{a})^2}{\varepsilon} \rightarrow \frac{y^2}{z^2} = \frac{(1 + \sqrt{a})^2}{\varepsilon} - \varepsilon$$

$$\frac{y^2}{z^2} = \frac{1 + a + 2\sqrt{a} - \varepsilon}{\varepsilon} \rightarrow \frac{y^2}{z^2} = \frac{2 + 2\sqrt{a}}{2} \rightarrow \frac{y^2}{z^2} = \frac{1 + \sqrt{a}}{2}$$

$$\rightarrow \frac{z^2}{y^2} = \frac{2}{1 + \sqrt{a}}$$

$$2a + \sqrt{2a^2 + \varepsilon a} = 2 \rightarrow \sqrt{2a^2 + \varepsilon a} = 2 - 2a$$

③

$$\frac{2}{\sqrt{a}} \rightarrow 2a^2 + \varepsilon a = 4a^2 + \varepsilon - 4a \rightarrow \sqrt{a^2 - 14a + \varepsilon} = 0$$

$$\sqrt{a^2 - 14a + \varepsilon} = 0 \rightarrow (a - 14)(a - \varepsilon) = 0 \rightarrow \begin{cases} a = 14 \\ a = \frac{\varepsilon}{2} \end{cases}$$

$$\frac{a+1}{a} = \frac{\frac{\varepsilon}{2} + 1}{\frac{\varepsilon}{2}} = \frac{2}{\varepsilon} = \frac{2}{\varepsilon} = \frac{2}{\varepsilon}$$

$$\frac{\sqrt{n+1}}{\sqrt{n-1} + c} - \frac{\sqrt{n+1}}{c - \sqrt{n-1}} = \frac{n-1}{\sqrt{n-1}} \quad (2)$$

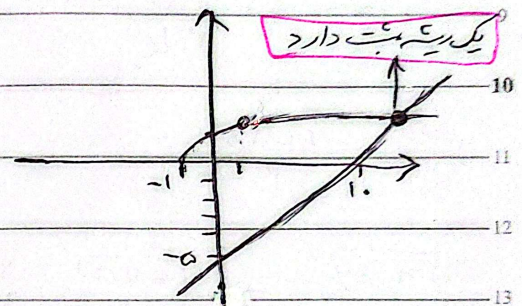
$$\frac{\sqrt{n+1} (c - \sqrt{n-1}) - \sqrt{n+1} (\sqrt{n-1} + c)}{c - (n-1)} = \frac{(\sqrt{n-1})^2}{\sqrt{n-1}}$$

$$\rightarrow \frac{c\sqrt{n+1} - \sqrt{n^2-1} - \sqrt{n^2-1} - c\sqrt{n+1}}{1-n} = \sqrt{n-1}$$

$$\rightarrow \frac{-2\sqrt{n^2-1}}{1-n} = \sqrt{n-1} \rightarrow -2\sqrt{n-1}\sqrt{n+1} = (1-n)\sqrt{n-1}$$

$\sqrt{n-1} \neq 0 \rightarrow n \neq 1$

$$-2\sqrt{n+1} = 1-n \rightarrow \sqrt{n+1} = \frac{n-1}{2}$$



$$\frac{1}{\sqrt{r-n} + r} - \frac{1}{r - \sqrt{r-n}} = \frac{r-n}{2\sqrt{r-n}} \quad (3)$$

$$\rightarrow \frac{r - \sqrt{r-n} - (\sqrt{r-n} + r)}{r - (r-n)} = \frac{r-n}{2\sqrt{r-n}} \rightarrow \frac{-2\sqrt{r-n}}{r+n} = \frac{r-n}{2\sqrt{r-n}}$$

$$\rightarrow -10(r-n) = r-n^2 \rightarrow -10 + 10n = r-n^2 \rightarrow n^2 + 10n - r = 0$$

$$\rightarrow (n-2)(n+12) = 0 \rightarrow n = -12 \text{ یا } n = 2$$

$n = 2$ جواب است. ندارد

$$\frac{1}{n^2} + \frac{1}{(1-n)^2} = \frac{19}{9} \rightarrow \left(\frac{1}{n} + \frac{1}{1-n}\right)^2 - 2\left(\frac{1}{n} + \frac{1}{1-n}\right) = \frac{19}{9}$$

$$\rightarrow \left(\frac{1}{n-n^2}\right)^2 - \frac{2}{n-n^2} = \frac{19}{9} \rightarrow \frac{1}{n-n^2} = t \rightarrow t^2 - 2t + 1 = \frac{19}{9} \rightarrow t = 1$$

$$(t-1)^2 = \frac{19}{9} \rightarrow t-1 = \pm \frac{\sqrt{19}}{3} \rightarrow t = \frac{1 \pm \sqrt{19}}{3}$$

$\frac{1}{n-n^2} = \frac{19}{9} \rightarrow -n^2 + n - \frac{5}{19} = 0 \Rightarrow S_1 = 1$

$\frac{1}{n-n^2} = \frac{-10}{9} \rightarrow -n^2 + n + \frac{10}{9} = 0 \Rightarrow S_2 = 1$

$$S_1 + S_2 = 2$$

$$1 \quad \sqrt{-x^2 + 4x - 1} \Rightarrow x^2 - 4x + 1 \leq 0 \rightarrow (x-2)(x-2) \leq 0 \quad \frac{2}{+} \frac{2}{-} \frac{1}{+} \rightarrow [2, 2] \quad (I)$$

$$3 \quad \sqrt{-x^2 + 4x + 1} \Rightarrow x^2 - 4x - 1 \leq 0 \rightarrow (x-2)(x+2) \leq 0$$

$$4 \rightarrow x(x-2) - 2(x+2) \leq 0 \rightarrow (x-2)(x+2) \leq 0$$

$$5 \quad \frac{-2}{-} \frac{2}{+} \frac{1}{-} \frac{1}{+} \quad (-\infty, -2] \cup [2, \infty) \quad (II) \quad (I) \cap (II) \rightarrow x=2$$

$$8 \quad \sqrt{4 + \sqrt{-4 + 4 + 1 - 1}} + \sqrt{19 + \sqrt{-19 + 19 - 1}} = 4 + 2 = 6 \quad \text{پاسخ داده شد}$$

$$10 \quad y = |x+2| + |x-1| \quad \begin{array}{c|c|c} -2 & -1 & 1 \\ \hline -x-2-x+1 & x+2-x+1 & x+2+x-1 \\ \hline -2x-1 & 3 & 2x+1 \end{array} \quad (A)$$

$$12 \quad \begin{cases} y + 2x = -1 \\ 3y + x = 14 \end{cases} \quad A \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} \quad \begin{cases} y - 2x = 1 \\ 3y + x = 14 \end{cases}$$

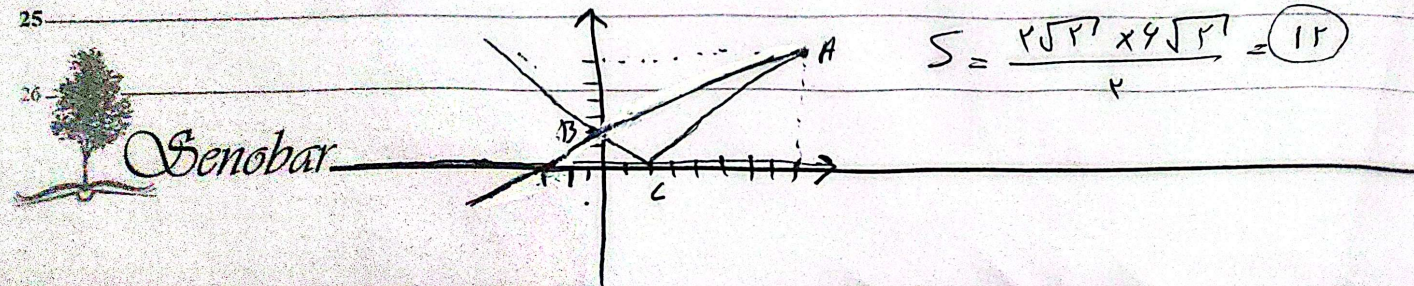
$$15 \quad x = \frac{-x+14}{3} \rightarrow x=1 \quad \text{و } y=2$$

$$17 \quad AB = \sqrt{(2x)^2 + (4y)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$

$$19 \quad \begin{cases} y + x = 2 \\ -y + \frac{x}{2} = -2 \end{cases} \quad \begin{cases} y - x = -2 \\ -y + \frac{x}{2} = -2 \end{cases} \quad \frac{3}{2}x = 0 \rightarrow x=0, y=2 \quad \frac{1}{2}x = -2 \rightarrow x=-4, y=2 \rightarrow A \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$$

$$22 \quad \sqrt{x^2 - 6x + 9} = \sqrt{(x-3)^2} = |x-3| \quad \frac{2}{-} \frac{2}{+} \frac{1}{-} \frac{1}{+}$$

$$24 \quad y = -x + 2 \quad \text{Area} = \frac{19 + 1 - 1}{\sqrt{2}} = \frac{19\sqrt{2}}{2} = 9\sqrt{2} \quad BC = \sqrt{9 + 4} = \sqrt{13}$$



روز $\rightarrow x = 14$

(10)

فرقا $\rightarrow x + 9$

$$\frac{1}{x} + \frac{1}{x+9} = \frac{1}{10} \rightarrow 10(x+9) + 10x = x(x+9)$$

$$10x + 110 + 10x = x^2 + 9x \rightarrow x^2 - 11x - 110 = 0$$

$$(x - 22)(x + 5) = 0$$

$$\boxed{x = 14}$$

$$x = 8 \quad 5.92$$