روژانو صدرالعلمائی — تکلیف ۱۸ — یازدهم دختر ۲

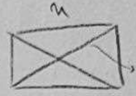
۱-

طول / عرض: $\frac{x}{y}=\frac{5}{4} \mapsto y=\frac{4x}{5}$

$\frac{x_2}{y}=\frac{1+\sqrt5}{2} \qquad x_2=\frac{1+\sqrt5}{2}y$

$$\frac{S_2}{S_1}=\frac{\frac{1+\sqrt5}{\cancel{2}}\cancel{y}\times \cancel{y}}{\frac{5}{\cancel{4}2}\cancel{y}\times \cancel{y}}=\boxed{0.4\,(1+\sqrt5)}$$

۲-



$$\frac{1+\sqrt5}{2}=\frac{\sqrt{x^2+y^2}}{x} \Rightarrow \frac{\overset{6}{\cancel{1+5}}+2\sqrt5}{4}=\frac{3+\sqrt5}{2}=\frac{x^2+y^2}{x^2} \to (1+\sqrt5)x^2=2y^2$$

$$\Rightarrow \frac{x^2}{y^2}=\frac{\cancel{2}(1-\sqrt5)}{\underbrace{(1+\sqrt5)(1-\sqrt5)}_{\cancel{-4}\ -2}}=\boxed{\frac{\sqrt5-1}{2}}$$

۳-

$3a+\sqrt{2a^2+4a}=2 \qquad \frac{a+1}{a}=? \longrightarrow a=\frac27 \longrightarrow \frac{\frac97}{\frac27}=\boxed{4.5}$

$\sqrt{2a^2+4a}=2-3a \xrightarrow{\text{توان ۲}}$

$2a^2+4a=4+9a^2-12a \longrightarrow 7a^2-16a+4=0 \longrightarrow a^2-16a+28=0 \quad (a-14)(a-2)=0$

$2=\frac{14}{7}$ یا $\frac27$

$a=2 \longrightarrow 6+\sqrt{16}=10$ ✗ $\qquad a=\frac27 \Rightarrow \frac67+\sqrt{2\times\frac{4}{49}+\frac{8\times 7}{7\times 7}}=\frac87+\frac67=2$ ✓

۴-

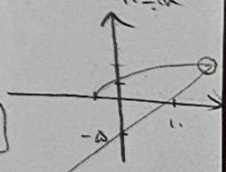
$$\frac{\sqrt{x+1}}{\sqrt{x-1}+3}-\frac{\sqrt{x+1}}{3-\sqrt{x-1}}=\frac{x-1}{\sqrt{\cancel{x-1}}}\ \ (\sqrt{x-1}\times\sqrt{x-1})$$

$(3+\sqrt{x-1})$

$$\longrightarrow \frac{\cancel{3\sqrt{x+1}}-\sqrt{(x+1)(x-1)}-\cancel{3\sqrt{x+1}}-\sqrt{(x-1)(x+1)}}{(3+\sqrt{x-1})(3-\sqrt{x-1})}=\frac{-2\sqrt{x^2-1}}{\underbrace{9-x+1}_{10-x}}$$

$$\longrightarrow \frac{-2\cancel{\sqrt{x-1}}\times\sqrt{x+1}}{10-x}=\cancel{\sqrt{x-1}} \longrightarrow \frac{-2\sqrt{x+1}}{10-x}=1 \longrightarrow \sqrt{x+1}=\frac{x}{2}-5$$

$\boxed{\text{یک ریشه مثبت دارد}}$



۵-

$$\frac{1}{\sqrt{2-x}+2}-\frac{1}{2-\sqrt{2-x}}=\frac{2-x}{5\cancel{\sqrt{2-x}}}\ \ \left(\cancel{(\sqrt{2-x})^2}\ \sqrt{2-x}\right)$$

$(2+\sqrt{2-x})$

$$\frac{\cancel{2}-\sqrt{2-x}-\cancel{2}-\sqrt{2-x}}{\underbrace{\cancel{4-2+x}}_{2+x}}=\frac{-2\sqrt{\cancel{2-x}}}{2+x}=\frac{\sqrt{\cancel{2-x}}}{5}$$

$-10=2+x$

$-8=x \rightarrow$ ریشه مثبت امکان ندارد.

۶-

$$\frac{1}{x^2}+\frac{1}{(1-x)^2}=\frac{140}{9} \qquad \frac{(1-x)^2+x^2}{\left(\underset{x-x^2}{x(1-x)}\right)^2}=\frac{140}{9}=\frac{1+x^2-2x+x^2}{(x^2-x)^2}=\frac{\overbrace{2x^2-2x}^{2(t)}+1}{(\underbrace{x^2-x}_{t})^2}$$

$$\frac{2t+1}{t^2}=\frac{140}{9} \qquad 140t^2=18t+9 \qquad t=\frac{18\pm\sqrt{\cancel{18^2+4\times140\times9}}^{\,78}}{280} \begin{cases} 0.3 \\ -\frac{3}{14}\end{cases} \qquad \begin{matrix} x^2-x-0.3=0 & S=1 \\ x^2-x+\frac{3}{14}=0 & S=1 \end{matrix} \; \boxed{=2}$$

۷-

$$\sqrt{x+\sqrt{-x^3+4x^2+25x-100}}+\sqrt{x^2+\sqrt{-x^2+6x-8}}\geq 0=x+2$$

$$-x^2(x-4)+25(x-4) \qquad -(x^2-6x+8)$$

$$(x-4)(25-x^2) \qquad -(x-4)(x-2)$$

$$(x-4)(5-x)(5+x)\geq 0$$

	2		4	
$-$		$+$		$-$

$$\sqrt{\cancel{4}}^{\,2}+\sqrt{\cancel{16}}^{\,4}\boxed{=6}$$

	-5		4		5	
$+$		$-$		$+$		$-$

$$D_f=(-\infty,-5]\cup[4,5]$$

$-5 \quad 2 \quad 4 \quad 5$ → تقاطع

۸-

$$y=|x+2|+|x-1| \qquad 3y+x=17 \rightarrow y=-\frac{1}{3}x+\frac{17}{3}$$

$\hookrightarrow -2 \qquad \hookrightarrow 1$

-2		1
$-2x-1=-\frac{1}{3}x+\frac{17}{3}$	$3=-\frac{1}{3}x+\frac{17}{3}$	$2x+1=-\frac{1}{3}x+\frac{17}{3}$
$-\frac{20}{3}=\frac{5}{3}x$	$\frac{1}{3}x=\frac{8}{3}$	$\frac{7}{3}x=\frac{14}{3}$
$x=-4$ ✓	$x=8$	$x=2$ ✓
$(-4,7)$	غ ق ق	$(2,5)$
B		A

$$AB=\sqrt{\Delta x^2+\Delta y^2}$$

$$=\sqrt{(\cancel{2+4})^2+(5-7)^2}=\sqrt{40}$$

$$\cancel{36}+\cancel{4}$$

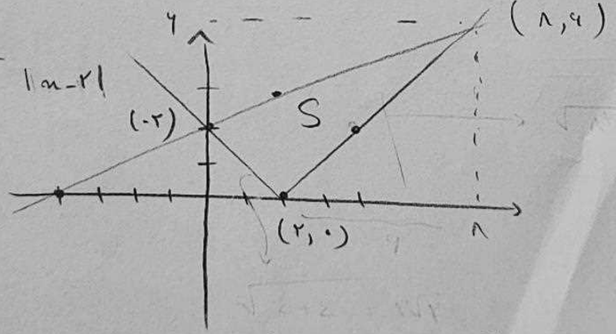
$$\boxed{2\sqrt{10}}$$

۹-

$$y=\sqrt{x^2-4x+4} \quad |x-2|$$

$$y=\frac{1}{2}x+2$$

-4	2
0	3



$$\frac{2}{2}x-2=\frac{1}{2}x+2$$

$$\frac{1}{2}x=4 \longrightarrow x=8 \longrightarrow y=6$$

$$\begin{pmatrix} 0 & 8 & 2 \\ 2 & 6 & 0 \\ 1 & 1 & 1 \end{pmatrix}=\left|\overset{-8}{4-12}\underbrace{-12-16}_{-24}\right|\times\frac{1}{2}\boxed{=12}$$

۱۰-

برادر خواهر

$$20x(x+9)\left(\frac{1}{x}+\frac{1}{x+9}=\frac{1}{20}\right) \quad 20(x+9+x)=x(x+9)$$

$$20(2x+9)=x^2+9x \longrightarrow x^2-31x-180=0 \qquad (x-36)(x+5)=0$$

$$40x+180. \qquad \hookrightarrow 36 \qquad \hookrightarrow -5 \times$$

برادر → $\boxed{36}$

خواهر → 45