باران کاظمی — یازدهم — تکلیف صفحه‌ی ۱۸

۱-

$x =$ طول

$y =$ عرض

$$\frac{x}{y} = \frac{5}{4} \qquad x = \frac{5}{4}y \qquad \frac{x+w}{y} = \frac{1+\sqrt5}{2}$$

$$\frac{5}{4}\cancel{\frac{x}{y}} + \frac{w}{y} = \frac{1+\sqrt5}{2} \Rightarrow \frac{w}{y} = \frac{2+2\sqrt5-5}{4} = \frac{2\sqrt5-3}{4}$$

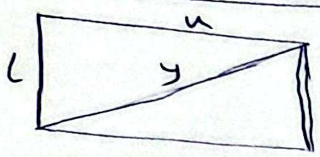
$$w = \left(\frac{2\sqrt5-3}{4}\right)y$$

$$xy = \frac{5}{4}y^2$$

$$(x+w)(y) = \Big(\underbrace{\tfrac{5}{4}y + \left(\tfrac{2\sqrt5-3}{4}\right)y}_{\left(\frac{2\sqrt5+\cancel{2}}{\cancel{4}\,2}\right)y}\Big)(y) = \frac{\sqrt5+1}{2}y^2$$

$$\rightarrow \frac{\frac{\sqrt5+1}{2}\cancel{y^2}}{2\,\frac{5}{\cancel{4}}\cancel{y^2}} = \boxed{\frac{2\sqrt5+2}{5}}$$

۲-



$$\frac{y}{x} = \frac{1+\sqrt5}{2} \qquad y = \left(\frac{1+\sqrt5}{2}\right)x$$

$$y^2 = l^2 + x^2 \Rightarrow \left(\left(\frac{1+\sqrt5}{2}\right)x\right)^2 = l^2 + x^2 \Rightarrow (3+\sqrt5)x^2 = l^2 + x^2$$

$$(2+\sqrt5)x^2 = l^2$$

$$\frac{x^2}{l^2} = \frac{1}{2+\sqrt5}_{\times\sqrt5-2} = \boxed{\sqrt5-2}$$

۳-

$$3a + \sqrt{2a^2+4a} = 2 \Rightarrow \sqrt{2a^2+4a} = 2-3a \Rightarrow 2a^2+4a = 4+9a^2-12a$$

$$7a^2 - 16a + 4 = 0 \rightarrow \frac{16 \pm \sqrt{256-112}}{14}\ {}^{12} = \frac{16\pm12}{14} \begin{cases} \frac{28}{14} = 2 \text{ غ ق ق} \\ \frac{4}{14} = \frac{2}{7} \checkmark \end{cases}$$

$2a^2+4a \geq 0$

$2a(a+2) \geq 0$

-2		0	
$+$	$-$		$+$

$1-3a \geq 0$

$a \leq \frac{1}{3}$

$$\frac{a+1}{a} = \frac{\frac{2}{7}+1}{\frac{2}{7}} = \frac{\frac{9}{\cancel7}}{\frac{2}{\cancel7}} = \boxed{\frac{9}{2}}$$

۴-

$$\left[\frac{\sqrt{x+1}}{\sqrt{x-1}+3} - \frac{\sqrt{x+1}}{3-\sqrt{x-1}} = \frac{x-1}{\sqrt{x-1}}\right] \times (\sqrt{x-1}+3)(3-\sqrt{x-1})(\sqrt{x-1})$$

$x > 1$

$$(\sqrt{x+1})\underbrace{(3-\sqrt{x-1})(\sqrt{x-1})}_{3\sqrt{x-1}-x+1} - (\sqrt{x+1})\underbrace{(\sqrt{x-1}+3)(\sqrt{x-1})}_{x-1+3\sqrt{x-1}} = (x-1)\underbrace{(\sqrt{x-1}+3)(3-\sqrt{x-1})}_{9-x+1 \;\; 10-x}$$

$$\cancel{3\sqrt{x^2-1}} - x\sqrt{x+1} + \sqrt{x+1} - x\sqrt{x+1} + \sqrt{x+1} - \cancel{3\sqrt{x^2-1}} = 10x - x^2 - 10 + x$$

$$-2x\sqrt{x+1} + 2\sqrt{x+1} = \underset{-(x^2-11x+10)}{-x^2+11x-10}$$

$$\sqrt{x+1}\,(2-2x) = -(x-10)(x-1) \qquad x \neq 1$$

$$\cancel{-}2\sqrt{x+1}\,(\cancel{x-1}) = \cancel{-}(x-10)(\cancel{x-1})$$

$$4(x+1) = x^2 - 20x + 100$$

$$4x + 4 = x^2 - 20x + 100 \Rightarrow x^2 - 24x + 96 = 0$$

$$x = \frac{24 \pm \sqrt{192}}{2} = 12 \pm 4\sqrt3$$

ق ق ($+$) — غ ق ق ($-$)

بقیه پشت

۵-

$$\frac{1}{2+\sqrt{2-x}}-\frac{1}{2-\sqrt{2-x}}=\frac{2-x}{5\sqrt{2-x}} \qquad \underbrace{\begin{matrix}2-x\geq 0\\ x\neq 2\end{matrix}}\quad x<2$$

$$\frac{\cancel{2}-\sqrt{2-x}-\cancel{2}-\sqrt{2-x}}{\frac{\cancel{4}-2+x}{2}}=\frac{2-x}{5\sqrt{2-x}} \qquad \frac{-2\sqrt{2-x}}{2+x}=\frac{2-x}{5\sqrt{2-x}}$$

ریشه ندارد

$x\leq 2$ غ ق ق

$x\leq 3$ غ ق ق

$$4-x^2=5(2-x) \Rightarrow 4-x^2=10-5x \Rightarrow x^2-5x+6=0$$
$$(x-3)(x-2)=0$$

۶-

$$\frac{1}{x^2}+\frac{1}{(1-x)^2}=\frac{160}{9}$$
$$\left(\frac{1}{x}+\frac{1}{1-x}\right)^2-2\frac{1}{x(1-x)}=\frac{160}{9}$$
$$\left(\frac{1-\cancel{x}+\cancel{x}}{x(1-x)}\right)^2-\frac{2}{x(1-x)}=\frac{160}{9}$$
$$\frac{1}{x(1-x)}=t$$
$$t^2-2t=\frac{160}{9} \Rightarrow t^2-2t-\frac{160}{9}=0$$
$$\frac{2\pm\sqrt{4+\frac{640}{9}}}{2}=\frac{2\pm\frac{26}{3}}{2}\begin{cases}\frac{32}{6}=\frac{16}{3}\\ -\frac{10}{3}\end{cases} \qquad \frac{676}{9}$$

$$\frac{1}{x(1-x)}=\frac{16}{3} \Rightarrow -x^2+x=\frac{3}{16} \Rightarrow x^2-x+\frac{3}{16}=0$$

مجموع ریشه ها $=1$

$$\frac{1}{x(1-x)}=\frac{-10}{3} \Rightarrow -x^2+x=-\frac{3}{10} \Rightarrow x^2-x-\frac{3}{10}=0$$

مجموع ریشه ها $=1$

$$1+1=2$$

۷-

$$\sqrt{x+\sqrt{-x^3+4x^2+25x-100}}+\sqrt{x^2+\sqrt{-x^2+6x-8}}=x+2$$

$$-x^2+6x-8\geq 0 \Rightarrow x^2-6x+8\leq 0$$
$$(x-4)(x-2)\leq 0 \qquad \begin{array}{c|c|c}2&&4\\ \hline +&-&+\end{array} \qquad \underline{2\leq x\leq 4}\ (1)$$

$$-x^3+4x^2+25x-100\geq 0 \Rightarrow -x^2(x-4)+25(x-4)\geq 0 \Rightarrow (x-4)\underbrace{(-x^2+25)}_{(5-x)(5+x)}\geq 0$$

$$\underline{(-\infty,-5]\cup[4,5]}\ (2) \qquad \begin{array}{c|c|c|c}-5&&4&5\\ \hline +&-&+&-\end{array}$$

$$(2)\cap(1) \longrightarrow x=4 \qquad \sqrt{4+\sqrt{0}}+\sqrt{16+\sqrt{0}}=2+4=6$$

$$\boxed{x+2=4+2=6} \qquad \boxed{x=4}$$

یک جواب دارد

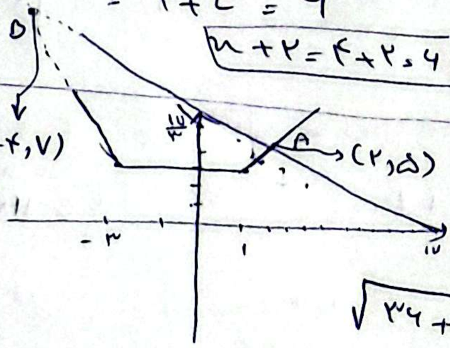
۸-

$$y=|x+2|+|x-1|$$
$$3y+x=17$$
$$y=-\frac{1}{3}x+\frac{17}{3}$$

x	y
0	$\frac{17}{3}$
17	0

$$\begin{array}{c|c|c}&-2&1\\ -2x-1&3&2x+1\end{array}$$

$$2x+1=-\frac{1}{3}x+\frac{17}{3}$$
$$\frac{7}{3}x=\frac{14}{3}$$
$$x=2$$



$$-2x-1=-\frac{1}{3}x+\frac{17}{3} \Rightarrow \frac{5}{3}x=-\frac{20}{3} \Rightarrow x=-4$$

$$\sqrt{36+4}=\sqrt{40}=\boxed{2\sqrt{10}}$$

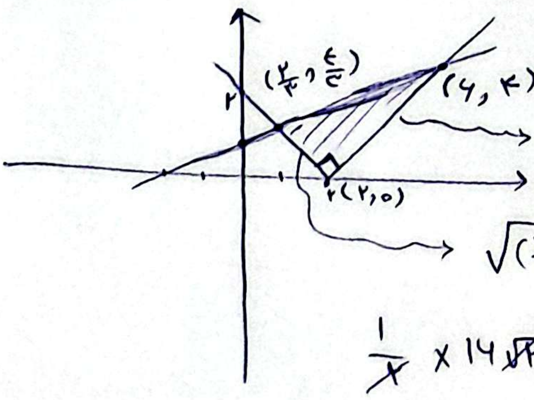
۹-

$$y=\sqrt{x^2-4x+4} \quad , \quad y=\frac{1}{2}x+1$$

x	y
0	1
-2	0

$$y=\sqrt{(x-2)^2}=|x-2|$$

$$|x-2|=\frac{1}{2}x+1 \begin{cases} x-2=\frac{1}{2}x+1 \\ \frac{1}{2}x=3 \Rightarrow \boxed{x=6}\ \boxed{y=4} \\ x-2=-\frac{1}{2}x-1 \\ \frac{3}{2}x=1 \Rightarrow \boxed{x=\frac{2}{3}}\ \boxed{y=\frac{4}{3}} \end{cases}$$



$$\sqrt{16+16}=4\sqrt{2}$$

$$\sqrt{\left(\frac{4}{3}\right)^2+\left(\frac{4}{3}\right)^2}=\frac{4}{3}\sqrt{2}$$

$$\frac{1}{\cancel{2}}\times 4\cancel{\sqrt{2}}\times\frac{4}{3}\cancel{\sqrt{2}}=\boxed{\frac{16}{3}}$$

۱۰ -

ساعت $x - 9 =$ فیروز

ساعت $x =$ فرهاد

$$\left[\frac{1}{x} + \frac{1}{x-9} = \frac{1}{20}\right] \times 20(x)(x-9)$$

$$= 20x - 180 + 20x = x^2 - 9x$$

$$x^2 - 49x + 180 = 0$$

$$(x-45)(x-4) = 0 \begin{cases} x = 4 \quad \text{غ ق} \quad x - 9 < 0 \\ x = 45 \ \checkmark \end{cases}$$

$$x - 9 = 45 - 9 = \boxed{36 \text{ ساعت}}$$