باران کاظمی   یازدهم   تکلیف صفحه‌ی ۱۸   ۱۷/۵

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$x =$ طول
$y =$ عرض

$\frac{x}{y} = \frac{5}{4}$

$x = \frac{5}{4}y$

$\frac{x+w}{y} = \frac{1+\sqrt5}{2}$

$\frac{5}{4}\cancel{\frac{x}{y}} + \frac{w}{y} = \frac{1+\sqrt5}{2} \Rightarrow \frac{w}{y} = \frac{2+2\sqrt5-5}{4} = \frac{2\sqrt5-3}{4}$

$w = \left(\frac{2\sqrt5-3}{4}\right)y$

$xy = \frac{5}{4}y^2$

$(x+w)(y) = \left(\underbrace{\frac{5}{4}y + \left(\frac{2\sqrt5-3}{4}\right)y}_{\left(\frac{2\sqrt5+2}{4}\right)y}\right)(y) = \frac{\sqrt5+1}{2}y^2$

$\rightarrow \frac{\frac{\sqrt5+1}{2}\cancel{y^2}}{2\times\frac{5}{4}\cancel{y^2}} = \boxed{\frac{2\sqrt5+2}{5}}$

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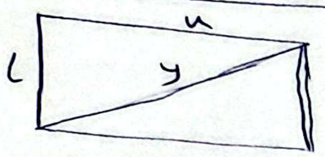

$\frac{y}{x} = \frac{1+\sqrt5}{2}$

$y = \left(\frac{1+\sqrt5}{2}\right)x$

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$y^2 = l^2 + x^2 \Rightarrow \left(\left(\frac{1+\sqrt5}{2}\right)x\right)^2 = l^2 + x^2 \Rightarrow \left(\frac{3+\sqrt5}{2}\right)x^2 = l^2 + x^2$

$(2+\sqrt5)x^2 = l^2$

$\frac{x^2}{l^2} = \frac{1}{2+\sqrt5}_{\times\sqrt5-2} = \boxed{\sqrt5-2}$

$\frac{\sqrt{x^2+y^2}}{x} = \frac{\sqrt5+1}{2} \rightarrow \frac{x^2+y^2}{x^2} = \frac{3+\sqrt5}{2}$

$l \quad \frac{y^2}{x^2} = \frac{1+\sqrt5}{2} \Rightarrow \frac{x^2}{y^2} = \frac{\sqrt5-1}{2}$

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$3a + \sqrt{2a^2+4a} = 2 \Rightarrow \sqrt{2a^2+4a} = 2-3a \Rightarrow 2a^2+4a = 4+9a^2-12a$

$7a^2 - 16a + 4 = 0 \rightarrow \frac{16 \pm \sqrt{\cancel{256-112}}\ 12}{14} = \frac{16\pm12}{14}$: $\frac{28}{14} = 2$ غ ق ق ، $\frac{4}{14} = \frac{2}{7}$ ✓

$2a^2+4a \geq 0$

$2a(a+2) \geq 0$

$\begin{array}{ccccc} & -2 & & 0 & \\ + & | & - & | & + \end{array}$

$1-3a \geq 0$

$a \leq \frac{1}{3}$

$\frac{a+1}{a} = \frac{\frac{2}{7}+1}{\frac{2}{7}} = \frac{\frac{9}{\cancel{7}}}{\frac{2}{\cancel{7}}} = \boxed{\frac{9}{2}}$

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$\left[\frac{\sqrt{x+1}}{\sqrt{x-1}+3} - \frac{\sqrt{x+1}}{3-\sqrt{x-1}} = \frac{x-1}{\sqrt{x-1}}\right] \times (\sqrt{x-1}+3)(3-\sqrt{x-1})(\sqrt{x-1})$

$x > 1$

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$(\sqrt{x+1})\underbrace{(3-\sqrt{x-1})(\sqrt{x-1})}_{3\sqrt{x-1}-x+1} - (\sqrt{x+1})\underbrace{(\sqrt{x-1}+3)(\sqrt{x-1})}_{x-1+3\sqrt{x-1}} = (x-1)\underbrace{(\sqrt{x-1}+3)(3-\sqrt{x-1})}_{9-x+1 \ = \ 10-x}$

$\cancel{3\sqrt{x^2-1}} - x\sqrt{x+1} + \sqrt{x+1} - x\sqrt{x+1} + \sqrt{x+1} - \cancel{3\sqrt{x^2-1}} = 10x - x^2 - 10 + x$

$-2x\sqrt{x+1} + 2\sqrt{x+1} = \underbrace{-x^2 + 11x - 10}_{-(x^2-11x+10)}$

$\sqrt{x+1}\,(2-2x) = -(x-10)(x-1)$ $\quad x \neq 1$

$\cancel{-}2\sqrt{x+1}\,(\cancel{x-1}) = \cancel{-}(x-10)(\cancel{x-1})$

$4(x+1) = x^2 - 20x + 100$

$4x+4 = x^2 - 20x + 100 \Rightarrow x^2 - 24x + 96 = 0$

$x = \frac{24 \pm \sqrt{192}}{2} = 12 \pm 4\sqrt3$

ق ق ($+$)

غ ق ق ($-$)

پذیرفته شد

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$$\frac{1}{2+\sqrt{2-x}}-\frac{1}{2-\sqrt{2-x}}=\frac{2-x}{5\sqrt{2-x}} \qquad \underbrace{2-x\geq 0}_{x\neq 2} \quad x<2$$

$$\frac{\cancel{2}-\sqrt{2-x}-\cancel{2}-\sqrt{2-x}}{\cancel{4}-2+x}=\frac{2-x}{5\sqrt{2-x}} \qquad \frac{-2\sqrt{2-x}}{2+x}=\frac{2-x}{5\sqrt{2-x}}$$

ریشه ندارد $x\leq 2$ ، $x=3$ غ ق ق

$$4-x^2=5(2-x)\Rightarrow 4-x^2=10-5x\Rightarrow x^2-5x+6=0 \quad (x-2)(x-3)=0$$

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$$\frac{1}{x^2}+\frac{1}{(1-x)^2}=\frac{140}{9}$$

$$\left(\frac{1}{x}+\frac{1}{1-x}\right)^2-2\,\frac{1}{x(1-x)}=\frac{140}{9}$$

$$\left(\frac{1-x+x}{x(1-x)}\right)^2-\frac{2}{x(1-x)}=\frac{140}{9}$$

$$\frac{1}{x(1-x)}=t \qquad t^2-2t=\frac{140}{9}\Rightarrow t^2-2t-\frac{140}{9}=0$$

$$\frac{2\pm\sqrt{4+\frac{560}{9}}}{2}=\frac{2\pm\frac{26}{3}}{2} \quad \frac{676}{9} \qquad \frac{32}{6}=\frac{16}{3} \qquad -\frac{10}{6}$$

$$\frac{1}{x(1-x)}=\frac{16}{3}\Rightarrow -x^2+x=\frac{3}{16}\Rightarrow x^2-x+\frac{3}{16}=0$$

مجموع ریشه‌ها $=1$

$$\frac{1}{x(1-x)}=-\frac{10}{6}\Rightarrow -x^2+x=-\frac{6}{10}\Rightarrow x^2-x-\frac{6}{10}=0$$

مجموع ریشه‌ها $=1$

$$1+1=2$$

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$$\sqrt{x+\sqrt{-x^3+4x^2+25x-100}}+\sqrt{x^2+\sqrt{-x^2+6x-8}}=x+2$$

$$-x^2+6x-8\geq 0\Rightarrow x^2-6x+8\leq 0 \quad (x-4)(x-2)\leq 0 \qquad 2\leq x\leq 4 \ (1)$$

$$-x^3+4x^2+25x-100\geq 0\Rightarrow -x^2(x-4)+25(x-4)\geq 0\Rightarrow (x-4)\underbrace{(-x^2+25)}_{(5-x)(5+x)}\geq 0$$

-5		4		5	
+	-		+		-

$$(-\infty,-5]\cup[4,5] \ (2)$$

$$(2)\cap(1)\rightarrow 4 \text{ یا } x=4 \quad \sqrt{4+\sqrt{0}}+\sqrt{16+\sqrt{0}}=2+4=6$$

$$x+2=4+2=6 \qquad \boxed{x=4}$$

یک جواب دارد.

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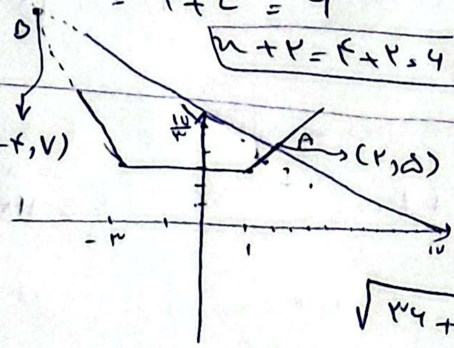
$$y=|x+2|+|x-1|$$

	-2		1	
$-2x-1$		3		$2x+1$

$$3y+x=17 \qquad y=-\frac{1}{3}x+\frac{17}{3}$$

x	y
0	$\frac{17}{3}$
17	0

$$2x+1=-\frac{1}{3}x+\frac{17}{3} \qquad \frac{7}{3}x=\frac{14}{3} \qquad x=2$$



$$-2x-1=-\frac{1}{3}x+\frac{17}{3}\Rightarrow -\frac{5}{3}x=-\frac{20}{3}\Rightarrow x=-4$$

$$\sqrt{36+4}=\sqrt{40}=\boxed{2\sqrt{10}}$$

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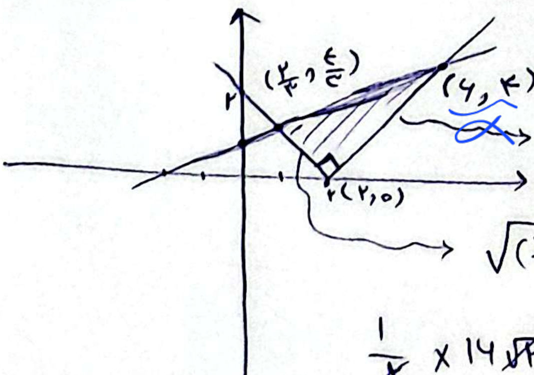
$$y=\sqrt{x^2-4x+4} \ , \ y=\frac{1}{2}x+1$$

x	y
0	1
-2	0

$$y=\sqrt{(x-2)^2}=|x-2|$$

$$|x-2|=\frac{1}{2}x+1 \begin{cases} x-2=\frac{1}{2}x+1 \quad \frac{1}{2}x=3\rightarrow \boxed{x=6} \ \boxed{y=4} \\ x-2=-\frac{1}{2}x-1 \quad \frac{3}{2}x=1\rightarrow \boxed{x=\frac{2}{3}} \ \boxed{y=\frac{4}{3}} \end{cases}$$



$$\sqrt{16+16}=4\sqrt{2}$$

$$\sqrt{\left(\frac{4}{3}\right)^2+\left(\frac{4}{3}\right)^2}=\frac{4}{3}\sqrt{2}$$

$$\frac{1}{\cancel{2}}\times 4\cancel{\sqrt{2}}\times\frac{4}{3}\cancel{\sqrt{2}}=\boxed{\frac{16}{3}}$$

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$$|x-2|=\frac{1}{2}x+2$$

$$A(0,2) \ B(2,0) \ C(8,6)$$

$$AB=2\sqrt{2} \qquad BC=6\sqrt{2} \qquad S=\frac{2\sqrt{2}\times 6\sqrt{2}}{2}=12$$

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فیروز $= x - 9$ ساعت

فرهاد $= x$ ساعت

$$\left[\frac{1}{x} + \frac{1}{x-9} = \frac{1}{20}\right] \times 20(x)(x-9)$$

$$= 20x - 180 + 20x = x^2 - 9x$$

$$x^2 - 49x + 180 = 0$$

$$(x - 45)(x - 4) = 0 \begin{cases} x = 4 \quad \text{غ ق ق} \quad x - 9 < 0 \\ x = 45 \ \checkmark \end{cases}$$

$$x - 9 = 45 - 9 = \boxed{36 \text{ ساعت}}$$

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