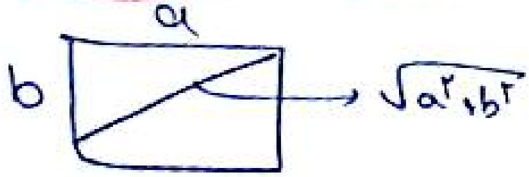
یازدهم دفتر ۱: C — به نام خدا — کار ریاضی، کلاس ۱۸

(۱)

$$\frac{S_{\text{طلایی}}}{S_{\text{اولیه}}}=\frac{\left(\left(\frac{1+\sqrt{5}}{2}\right)\times 4x\right)\times 4x}{5x\times 4x}=\frac{2+2\sqrt{5}}{5}=0.4+0.4\sqrt{5}=0.4(1+\sqrt{5})$$

(۲)



$$\frac{\sqrt{a^2+b^2}}{a}=\frac{1+\sqrt{5}}{2}$$

به توان ۲:

$$\frac{a^2+b^2}{a^2}=\left(\frac{1+\sqrt{5}}{2}\right)^2$$

$$\rightarrow 1+\frac{b^2}{a^2}=\frac{6+2\sqrt{5}}{4}\rightarrow \left(\frac{b}{a}\right)^2=\frac{6+2\sqrt{5}}{4}-1$$

$$\left(\frac{b}{a}\right)^2=\frac{2+2\sqrt{5}}{4}=\frac{1+\sqrt{5}}{2}$$

$$\Rightarrow \left(\frac{a}{b}\right)^2=\frac{2}{1+\sqrt{5}}$$

(۳)

$$3a+\sqrt{2a^2+4a}=2\rightarrow\sqrt{2a^2+4a}=2-3a\xrightarrow{\text{توان ۲}}2a^2+4a=4-12a+9a^2$$

$$7a^2-16a+4=0\rightarrow\Delta=(-16)^2-4(7)(4)=144\rightarrow\sqrt{\Delta}=12$$

$$a=\frac{16\pm 12}{14}\rightarrow a=2\rightarrow \times$$

$$a=\frac{2}{7}\rightarrow\frac{a+1}{a}=1+\frac{1}{a}=1+\frac{7}{2}=4{,}5$$

④

$$\frac{\sqrt{x+1}}{\sqrt{x-1}+3}-\frac{\sqrt{x+1}}{3-\sqrt{x-1}}=\frac{x-1}{\sqrt{x-1}}$$

$$\sqrt{x+1}\left(\frac{3-\sqrt{x-1}-3-\sqrt{x-1}}{(3+\sqrt{x-1})(3-\sqrt{x-1})}\right)=\sqrt{x-1}$$

$$\sqrt{x+1}\left(\frac{-2\sqrt{x-1}}{9-(x-1)}\right)=\sqrt{x-1}\Rightarrow\frac{-2\sqrt{x+1}}{10-x}=1$$

$$+2\sqrt{x+1}=x-10$$

$$4(x+1)=x^2-20x+100$$

$$x^2-24x+96=0\longrightarrow\Delta=(-24)^2-4\times96=192\rightarrow x=\frac{24\pm\sqrt{192}}{2}=$$

$$=12\pm4\sqrt{3}$$

غ ق ق ← $12-4\sqrt{3}$

جواب $\Rightarrow$ $\boxed{12+4\sqrt{3}}$

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⑤

$$\frac{1}{\sqrt{2-x}+2}-\frac{1}{2-\sqrt{2-x}}=\frac{2-x}{5\sqrt{2-x}}$$

$$\frac{2-\sqrt{2-x}-\sqrt{2-x}-2}{(2+\sqrt{2-x})(2-\sqrt{2-x})}=\frac{2-x}{5\sqrt{2-x}}$$

$$\frac{-2\sqrt{2-x}}{4-(2-x)}=\frac{2-x}{5\sqrt{2-x}}\Rightarrow-10(2-x)=(2+x)(2-x)$$

$$-20+10x=4-x^2$$

$$x^2+10x-24=0\rightarrow(x+12)(x-2)=0$$

$x=-12$, $x=2$ → غ ق ق

معادله جواب ندارد ←

$$\frac{1}{x^2}+\frac{1}{(1-x)^2}=\frac{160}{9} \rightarrow \frac{(1-x)^2+x^2}{x^2(1-x)^2}=\frac{160}{9}$$

$$\frac{2x^2-2x+1}{(x-x^2)^2}=\frac{160}{9}$$

$$\frac{2(x^2-x)+1}{(x^2-x)^2}=\frac{160}{9} \xrightarrow{x^2-x=t} \frac{2t+1}{t^2}=\frac{160}{9}$$

$$160t^2=18t+9 \rightarrow 160t^2-18t-9=0 \rightarrow t=\frac{18\pm\sqrt{(-18)^2-4(160)(-9)}}{2\times 160}$$

$$t=\frac{18\pm 6\sqrt{169}}{2\times 160} \Rightarrow \begin{cases} t=-\frac{3}{16} \Rightarrow x^2-x=-\frac{3}{16} \rightarrow x^2-x+\frac{3}{16}=0 \\ t=\frac{3}{10} \Rightarrow x^2-x=\frac{3}{10} \Rightarrow x^2-x-\frac{3}{10}=0 \end{cases}$$

$$\boxed{1+1=2}$$

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(۷)

$$\sqrt{x+\sqrt{-x^3+4x^2+25x-100}}+\sqrt{x^2+\sqrt{-x^2+6x-8}}=x+2$$

$-x^2+6x-8\geq 0 \rightarrow x^2-6x+8\leq 0 \rightarrow (x-2)(x-4)\leq 0 \rightarrow 2\leq x\leq 4$ (I)

$-x^3+4x^2+25x-100\geq 0 \Rightarrow x^2(-x+4)+25(x-4)\geq 0 \rightarrow (x-4)(25-x^2)\geq 0$

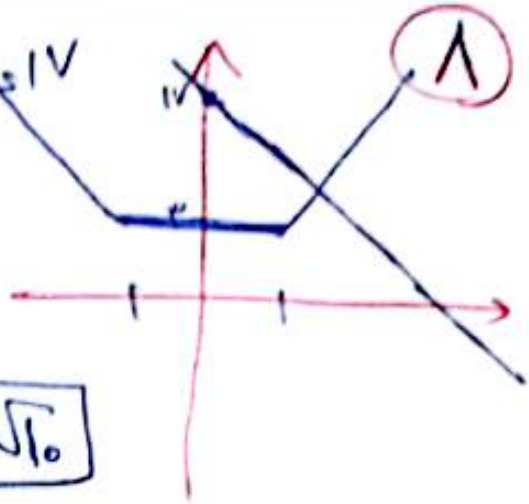
$x\leq -5$ یا $4\leq x\leq 5$ (II)

$(II)\cap(I) \Rightarrow x=4 \rightarrow \sqrt{4+\sqrt{0}}+\sqrt{16+\sqrt{0}}=4+2 \Rightarrow 2+4=4+2$ درست

پس $\boxed{x=4}$

۸

$y=|x+2|+|x-1|$



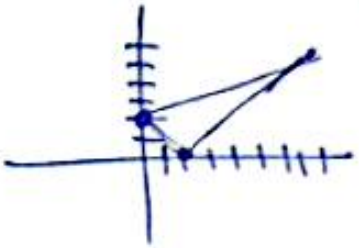
$A\begin{vmatrix}2\\5\end{vmatrix}$ $B\begin{vmatrix}-4\\7\end{vmatrix}$

$AB=\sqrt{(2-(-4))^2+(5-7)^2}=\sqrt{40}=2\sqrt{10}$

۹

$y=\frac{1}{2}x+2$

$y=\sqrt{x^2-4x+4} \Rightarrow \sqrt{(x-2)^2}$



$y=|x-2|$ $\quad |x-2|=\frac{1}{2}x+2$

$y=4 \Leftrightarrow x=1 \leftarrow x-2=\frac{1}{2}x+2$

$y=2,\ x=0 \leftarrow x-2=-\frac{1}{2}x-2$

$A\begin{vmatrix}0\\2\end{vmatrix}$ $B\begin{vmatrix}2\\0\end{vmatrix}$ $C\begin{vmatrix}4\\2\end{vmatrix}$

$AB=2\sqrt{2}$

$BC=2\sqrt{2}$ $\quad S=\frac{2\sqrt{2}\times 2\sqrt{2}}{2}=12$

۱۰

کمرز $\Rightarrow x \rightarrow \frac{1}{x}$

تنها $\Rightarrow x+9 \rightarrow \frac{1}{x+9} \Rightarrow \frac{1}{x}+\frac{1}{x+9}=\frac{1}{20}$

$\frac{x+x+9}{x(x+9)}=\frac{1}{20} \Rightarrow \frac{2x+9}{x^2+9x}=\frac{1}{20} \rightarrow 40x+180=x^2+9x$

$x^2-31x=180$

$x^2-31x-180=0$

$x=\frac{+31\pm\sqrt{(-31)^2-4\times(-180)}}{2} \rightarrow \sqrt{961}=31$

$x=36 \rightarrow$ ✓

$x=-5 \rightarrow$ نمی‌تواند